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The war of attrition and the revelation of valuable information[☆]

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HIGHLIGHTS

- The revelation information in a war of attrition with private budget constraints can decrease expected revenue.
- Information non-revelation can counteract the adverse revenue impact of budget constraints.
- The linkage principle is sensitive to the presence of private budget constraints.

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ABSTRACT

We provide a simple example demonstrating that the unconditional revelation information in a war of attrition with private budget constraints can decrease expected revenue. Our example suggests that information non-revelation can counteract the adverse revenue impact of budget constraints and almost counterbalance their otherwise negative impact.

Milgrom and Weber (1982) establish that an auctioneer should unconditionally reveal relevant privately-held information. In expectations such a policy cannot adversely affect revenue. This result is often noted as a case of the *linkage principle*. Building on Fang and Parreiras (2002), Fang and Parreiras (2003) construct an example of a second-price auction where the linkage principle holds if bidders are mildly budget-constrained, but fails once those constraints are sufficiently severe. We provide a new example demonstrating the drawbacks of information revelation when agents are budget constrained, even if those constraints are arbitrarily mild.

Our example focuses on the war of attrition, which is of interest for several reasons.[1] When agents are budget-constrained, allocation schemes with an all-pay flavor are generally considered to be revenue-superior to their "winner-only-pay" counterparts, such as first- or second-price auctions (Maskin, 2000; Pai and Vohra, 2014). Thus, our study is a preliminary step toward incorporating information disclosure into the study of optimal mechanisms with budget-constrained bidders.[2] We argue that a policy of non-disclosure can sometimes almost fully counteract the adverse revenue implications of budget constraints. Thus, information disclosure is possibly a strong policy lever in these environments. Additionally, our analysis identifies a tractable class of examples for the study of auctions with private budget constraints, which may aid others studying related questions.

Our note is organized simply. The next section introduces our model, but without budget constraints. Section 2 incorporates private budget constraints and examines revenue under disclosure and non-disclosure regimes.

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[1] We model the war of attrition as a static game, as in Krishna and Morgan (1997).

[2] Gershkov (2009) shows that in an environment without budget constraints, an optimal selling mechanism involves full information disclosure.

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1. A war of attrition

Consider a war of attrition with two ex ante symmetric bidders competing for a single prize.[3] Each bidder i submits a bid, $b_i \geq 0$, and the highest bidder is the winner. If bidder i is the winner, his payoff is $V_i - b_j$ where V_i is his valuation and b_j is the bid of bidder $j \neq i$. Bidder j's payoff in this scenario is $-b_j$. A fair coin flip resolves ties.

Bidder i's valuation is a product of two pieces of information: $V_i = S_i S_0$. S_i is bidder i's private signal concerning the prize's value. It is known only to him and is unknown to others. S_0 is information held by a third party (the "seller"). It is initially unknown to either bidder, but may become publicly known depending on the disclosure regime. For all i, S_i is distributed independently according to the cumulative distribution function (c.d.f.) $F(s)\colon [0, \bar{s}] \to [0, 1]$ with corresponding density $f(s) > 0$. The seller's information is such that $\Pr[S_0 = 1] = p$ and $\Pr[S_0 = 0] = 1 - p$. All prior distributions are common knowledge.

A motivation for this setup is a patent race. S_i is firm i's private benefit from securing the patent. S_0 describes the patent authority's information about the new good's patentability or the authority's willingness to enforce a patent. If the good is not patentable, the invention is worthless.

The seller can adopt one of two disclosure policies. Under a policy of *no disclosure* (N) the seller commits to not informing the bidders of the realized value of S_0 prior to bidding. Under a policy of *disclosure* (D), the seller always informs the bidders of the realized value of S_0 prior to bidding.[4] In each case, the equilibrium bidding functions defined below follow from Krishna and Morgan (1997).

No disclosure. The expected value of the prize to bidder i in a non-disclosure regime is $v_N(s) = \mathbb{E}[S_i S_0 | S_i = s] = ps$. Thus, the symmetric equilibrium bidding strategy is

$$b_N(s) = \int_0^s v_N(y)\frac{f(y)}{1-F(y)}dy = \int_0^s py\frac{f(y)}{1-F(y)}dy.$$

Given two independent random variables each with c.d.f. $F(s)$, $H(s) = F(s)^2 + 2(1-F(s))F(s)$ is the c.d.f. of the second-highest order statistic. Let $h(s)$ be the associated density. Therefore, the expected revenue is

$$R_N = 2 \times \int_0^{\bar{s}} b_N(s)h(s)ds = 2p\int_0^{\bar{s}}\int_0^s \frac{yf(y)}{1-F(y)}dy\, h(s)\, ds.$$

Disclosure. Suppose instead that the seller adopts a policy of unconditional disclosure. If $S_0 = 0$, the prize's value is zero and all bidders bid zero. If instead $S_0 = 1$, the prize's value is $v_D(s) = \mathbb{E}[S_i S_0 | S_i = s, S_0 = 1] = s$. Thus, a bidder with private signal s bids

$$b_D(s) = \int_0^s v_D(y)\frac{f(y)}{1-F(y)}dy = \int_0^s y\frac{f(y)}{1-F(y)}dy.$$

Therefore, the expected revenue is

$$\begin{aligned} R_D &= 2 \times \left[p \times \int_0^{\bar{s}} b_D(s)h(s)ds + (1-p)\times 0\right] \\ &= 2p\int_0^{\bar{s}}\int_0^s \frac{yf(y)}{1-F(y)}dy\, h(s)\, ds. \end{aligned}$$

Both regimes yield the same revenue and the seller is indifferent between disclosure or concealment of private information.

Example 1. Suppose $\bar{s} = 1$ and $F(s) = s$. Thus, bidders' types are uniformly distributed. In this case, $R_N = R_D = p/3$.

2. A war of attrition with budget constraints

Expanding our model, suppose additionally that bidders face a private budget constraint, as in Che and Gale (1998) or Kotowski (2013). A bidder's private information consists of his signal S_i, as above, and a private budget, W_i. A bidder with a budget of $W_i = w$ must bid less than w. Private budget constraints feature in many applications. For example, in a patent race budget constraints stem from firms' financial capacity or by budgetary allocations to research groups (Leininger, 1991). S_i and W_i are independent and each W_i is independently distributed according to the c.d.f. $G(w)$ with density $g(w)$.

Kotowski and Li (2014) study the war of attrition with private budget constraints. They show that, under appropriate regularity conditions, there exists a symmetric equilibrium in nondecreasing, continuous strategies of the form $\beta(s, w) = \min\{\hat{b}(s), w\}$. If $v(s)$ denotes the prize's expected value given the available information, then $\hat{b}(s)$ is a solution of the differential equation

$$\hat{b}'(s) = \frac{(1-G(\hat{b}(s)))v(s)f(s)}{(1-G(\hat{b}(s)))(1-F(s)) - g(\hat{b}(s))(1-F(s))v(s)} \tag{1}$$

subject to an appropriate boundary condition.[5]

Except in special cases, as in the continuation of Example 1, (1) rarely admits an elegant solution. Moreover, each bidder's strategy β is a function of his multi-dimensional type. Thus, to compute revenues we first define a distribution of "pseudo-types" who (given the distribution of budgets and the equilibrium bidding strategy) bid less than $\hat{b}(s)$. We use this distribution to compute revenues.[6]

No Disclosure. Let $\hat{b}_N(s)$ be the solution to (1) when $v(s) = v_N(s) = ps$. Given the strategy $\beta_N(s, w) = \min\{\hat{b}_N(s), w\}$, the probability that a bidder with signal $S_i = s$ bids less than $\hat{b}_N(s)$ is $\hat{F}_N(s) = F(s) + (1-F(s))G(\hat{b}_N(s))$. If $\hat{H}_N(s) = \hat{F}_N(s)^2 + 2(1-\hat{F}_N(s))\hat{F}_N(s)$, then the expected revenue generated by the non-disclosure regime is $\hat{R}_N = 2 \times \int_0^{\bar{s}} \hat{b}_N(s)\hat{h}_N(s)ds$.

Disclosure. As above, both bidders bid zero when $S_0 = 0$. Thus, let $\hat{b}_D(s)$ be the solution to (1) when $v(s) = v_D(s) = s$, i.e. it is known that $S_0 = 1$. Defining terms analogously to the previous paragraph, the expected revenue is $\hat{R}_D = 2 \times [p\int_0^{\bar{s}} \hat{b}_D(s)\hat{h}_D(s)ds]$.

Example 1 (*Continued*)**.** Continuing to assume that $F(s) = s$, suppose additionally that $G(w) = 1 - e^{-w/\alpha}$, $\alpha \geq 1$, is the distribution of budgets. Solving (1) with $v(s) = v_N(s) = ps$ and the boundary condition $\hat{b}_N(0) = 0$ gives

$$\hat{b}_N(s) = \frac{\alpha^2 \log\left(1 - \frac{ps}{\alpha}\right) - \alpha p \log(1-s)}{\alpha - p}. \tag{2}$$

(Appendix A presents a short verification that $\beta_N(s, w) = \min\{\hat{b}_N(s), w\}$, as defined above, constitutes an equilibrium in

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[3] For expositional ease we adopt a phrasing from auction theory. Agents are bidders, actions are bids, etc.

[4] The subscript "N" denotes values associated with a non-disclosure regime. The subscript "D" is used in the case of disclosure.

[5] Kotowski and Li (2014) outline how to identify the boundary condition. For the purposes of the present paper, the appropriate boundary condition is $\hat{b}(0) = 0$.

[6] Our approach to compute revenues depends on a single-dimensional re-parameterization of our model. See Araujo et al. (2008) for a general discussion of re-parameterizations of auction models where agents have multi-dimensional types. Che and Gale (1998) and Che and Gale (2006) employ variations of similar procedures to compute revenues.