Economics Letters 124 (2014) 457–460

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

The unique equilibrium in a model of sales with costly advertising

Michael A. Arnold [a,1], Lan Zhang [b,*]

[a] Department of Economics, Alfred Lerner College of Business and Economics, University of Delaware, Newark, DE, 19806, USA

[b] Research Institute of Economics and Management, Southwestern University of Finance and Economics, Chengdu, China

HIGHLIGHTS

- We incorporate costly advertising in Varian's (1980) model of sales.
- We show that a unique equilibrium exists, and that this equilibrium is symmetric.
- In contrast, multiple asymmetric equilibria exist with no advertising cost.
- The equilibrium becomes less competitive as the advertising cost increases.

ARTICLE INFO

Article history:
Received 31 March 2014
Received in revised form
24 June 2014
Accepted 7 July 2014
Available online 12 July 2014

JEL classification:
D4
D83
L19
L89

Keywords:
Advertising
Sales
Unique equilibrium
Price dispersion

ABSTRACT

We demonstrate that the Varian (1980) model of sales has a unique Nash equilibrium when firms incur costly advertising to compete for informed consumers. The equilibrium is symmetric. In particular, with costly advertising, the asymmetric equilibria highlighted by Baye et al. (1992) do not arise.

1. Introduction

Varian's (1980) model of sales has served as a foundation for a large literature exploring temporal equilibrium price dispersion.[2] Varian derives a symmetric equilibrium in which sales arise randomly as the result of mixed pricing strategies by firms competing in a market with informed and uninformed consumers.

Baye et al. (1992) extend Varian's results to show that in addition to the symmetric equilibrium, a continuum of asymmetric equilibria also exist. In these asymmetric equilibria some firms randomize over the interval $[\underline{p}, r]$, where $\underline{p}$ is the lowest price any firm might post and r is the reservation price common to all buyers, and other firms randomize over intervals $[\underline{p}, x_i]$, where $x_i < r$, and place positive mass on r. They then show that these asymmetric equilibria can be ranked by first-order stochastic dominance and that in a metagame played by both buyers and sellers, Varian's symmetric equilibrium emerges as the unique subgame perfect equilibrium.

Baye and Morgan (2001) introduce an advertising cost incurred by firms competing for informed consumers in a framework similar to that considered by Varian. They derive a symmetric equilibrium outcome of the resulting game but do not consider the possibility of asymmetric equilibria. In this paper, we demonstrate that there is a unique Nash equilibrium in the Varian model of sales with positive

* Corresponding author.
E-mail addresses: marnold@udel.edu (M.A. Arnold), zhanglan@swufe.edu.cn (L. Zhang).
1 Tel.: +1 302 831 2340.

2 A literature that slightly preceded Varian's work focused on spatial price dispersion in which some firms persistently charge lower prices than others (e.g., Salop and Stiglitz, 1977, Butters, 1977 and Pratt et al., 1979). Varian presents a framework in which firms randomly set prices, so the price charged by a given firm fluctuates over time. See Baye et al. (2006) for a survey of this literature.

http://dx.doi.org/10.1016/j.econlet.2014.07.005

advertising costs. In particular, asymmetric equilibria that exist in the Varian framework do not arise if advertising is costly.

To understand the relationship between our main result and the asymmetric equilibria found by Baye, Kovenock and de Vries, note that in their framework advertising is not possible. Placing positive mass on r is effectively a decision to be less competitive for informed consumers. The metagame they consider has a unique subgame perfect equilibrium because the asymmetric equilibria entail price dispersion in which some firms charge consistently higher prices than others—a property Varian noted is not sustainable. In our setting, informed consumers obtain price information from firms that engage in costly advertising. Advertising a price of r reflects conflicting decisions to be less competitive for informed consumers and to simultaneously incur the advertising cost. As we demonstrate below, this conflict eliminates the asymmetric equilibria without requiring subgame perfection; the model of sales with costly advertising has a unique symmetric Nash equilibrium.

2. A model of sales with costly advertising

Following Varian (1980), consider a market with two types of consumers, informed and uninformed. Let $I > 0$ be the number of informed consumers, $M > 0$ be the number of uninformed consumers, and $n \geq 2$ be the number of firms, so $U = M/n$ is the number of uninformed consumers per store. All consumers have a reservation price r for one unit of the product. There is a single price information clearinghouse (such as a newspaper or price-comparison website) in the market. Each firm may choose to advertise its price through this site at a cost of $\phi > 0$. Informed consumers always access advertised prices and purchase from the firm with the lowest advertised price $p \leq r$. If several firms advertise the same lowest price, then informed consumers are equally allocated across these firms. If no firms advertise, then informed consumers randomly choose a firm and purchase from that firm if $p \leq r$. Uninformed consumers do not access price information. They are equally allocated across the n firms and purchase if $p \leq r$. Following earlier literature, we assume firms cannot price discriminate across informed and uninformed consumers.[3]

Define the advertising strategy α_i as the probability that firm i advertises, and the pricing strategy $F_i(p)$ which denotes, conditional on the decision to advertise, the probability that firm i advertises a price $p_i \leq p$. (Clearly a firm that does not advertise maximizes profit by charging $p = r$.) Firms simultaneously determine advertising and pricing strategies and utilize these strategies to make advertising and pricing decisions. To simplify the presentation, we assume that firms produce the homogeneous good at a constant marginal cost c which, without loss of generality, we set to $c = 0$. Our results also hold with a more general cost function $C(q)$ as in Varian and in Baye, Kovenock and de Vries (see footnote 4 below for a brief explanation).

Firm i's expected profit $\pi_i^A(p)$ from advertising a price of p is

$$\pi_i^A(p) = pU + pI \prod_{j \neq i} \left(1 - \alpha_j F_j(p)\right) - \phi$$

and expected profit π_i^N from not advertising and setting $p = r$ is

$$\pi_i^N = rU + r\frac{I}{n} \prod_{j \neq i} \left(1 - \alpha_j\right).$$

Note that at least two firms must advertise with positive probability in equilibrium. If not, then one firm i would advertise $p_i = r$ with probability $\alpha_i = 1$ and capture all of the informed consumers. But then another firm j would have an incentive to advertise $p_j = r - \varepsilon$.

We now state, without proof, several properties of $F_i(p)$ established in earlier literature which also apply in our setting. Note that there can be no interval that is in the support of only one firm's pricing strategy, and any price in the interval from the lowest price $\underline{p}$ advertised by any firm to the reservation price r must be in the supports of at least two firms' pricing strategies. This follows from the fact that if the interval $[p, p + \varepsilon]$ is in the support of only firm's pricing strategy, then that firm's expected profit would be strictly increasing in p on this interval. Similarly, $F_i(p)$ can have no mass points except, possibly, at $p_i = r$, and the upper support $\bar{p}_i$ of at least two firms' pricing strategies must be r. These results follow from standard undercutting arguments and the fact that if $\bar{p}_i < r$ for all i, then for the firm with the highest upper support profit would be strictly increasing over the interval $[\bar{p}_i, r]$.

Baye, Kovenock and de Vries demonstrate that with no advertising cost, equilibria in which several firms have a mass point at r can occur. Below we show that if advertising is costly, it is never optimal for a firm to adopt a pricing strategy that places positive mass at r. It will be useful to define

$$\tilde{p}_i = \frac{r\left(U + \frac{I}{n}\prod_{j \neq i}\left(1 - \alpha_j\right)\right) + \phi}{U + I}, \tag{1}$$

which is the price at which a firm that advertises and sells to the I informed consumers plus its share U of uninformed consumers obtains the same profit that it would obtain by not advertising and charging the reservation price r. Note that firm i would never advertise a price less than $\tilde{p}_i$ because a lower price would yield less profit than could be obtained by not advertising and setting $p = r$. Let $\underline{p}_i \geq \tilde{p}_i$ denote the lower support of firm i's pricing strategy, and let $\underline{p} = \min\left\{\underline{p}_i\right\}$. We now establish several lemmas which incorporate the role of costly advertising and the advertising probability α_i.

Lemma 1. *In equilibrium no firm can advertise with probability* $\alpha_i = 1$.

Proof. First suppose $\alpha_i = \alpha_j = 1$ for two firms i and j. Let $\bar{p}_k$ denote the upper support of firm k's pricing strategy for $k = i, j$. Note that $\bar{p}_i = \bar{p}_j = \bar{p}$. To see this, suppose $\bar{p}_i > \bar{p}_j$. Because $\alpha_j = 1$ and $F_j(\bar{p}_j) = 1$, firm i would never sell to informed consumers at any $p \in [\bar{p}_j, \bar{p}_i]$. Thus, firm i should either increase its price to $\bar{p}_i$ and not advertise (which contradicts $\alpha_i = 1$), or should redistribute mass in this interval to $p \leq \bar{p}_j$. Similar arguments imply $\bar{p}_j > \bar{p}_i$ is not possible.

Because $\alpha_i = \alpha_j = 1$, for $k = i, j$ equilibrium expected returns must satisfy $\pi_k^A(p) \geq \pi_k^N$ for all prices in the support of each firm's pricing strategy. If neither firm has a mass point at $\bar{p}$ or if firm i but not firm j has a mass point at $\bar{p}$ (recall that standard undercutting arguments imply that at most one firm can have a mass point at $\bar{p}$ and this can only occur if $\bar{p} = r$), then because $F_j(\bar{p}) = 1$ and $\alpha_j = 1$,

$$\pi_i^A(\bar{p}) - \pi_i^N = \bar{p}U - \phi - rU \leq -\phi < 0$$

which contradicts $\pi_i^A(\bar{p}) \geq \pi_i^N$, so $\alpha_i = 1$ is not optimal. Thus, at most one firm can advertise with probability 1 in equilibrium.

To verify that $\alpha_i = 1$ cannot occur, suppose $\alpha_i = 1$, so $\alpha_j < 1$ for all $j \neq i$. From Eq. (1) we have $\tilde{p}_j = (rU + \phi)/(U + I) < \left(r\left(U + \frac{I}{n}\Pi_{k \neq i}(1 - \alpha_k)\right) + \phi\right)/(U + I) = \tilde{p}_i$ for all $j \neq i$ where the inequality follows from $\alpha_j < 1$ for all $j \neq i$. This implies $\tilde{p}_i > \underline{p}$ because $\tilde{p}_i \leq \underline{p}$ implies $\tilde{p}_j < \underline{p}$ which implies $\alpha_j = 1$ is optimal for

[3] See Baye and Morgan (2002) for a model in which firms can price discriminate.