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Tournaments with gaps[☆]

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HIGHLIGHTS

- We analyze a moral-hazard problem with two risk averse agents.
- Since performance is unverifiable a tournament is used as a credible incentive scheme.
- Standard tournament contracts specify only tournament prizes.
- We show how this standard tournament can be modified to reduce labor costs.
- Such reduction is possible under unlimited liability but not under limited liability.

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ABSTRACT

A standard tournament contract specifies only tournament prizes. If agents' performance is measured on a cardinal scale, the principal can complement the tournament contract by a gap which defines the minimum distance by which the best performing agent must beat the second best to receive the winner prize. We analyze a tournament with two risk averse agents. Under unlimited liability, the principal strictly benefits from a gap by partially insuring the agents and thereby reducing labor costs. If the agents are protected by limited liability, the principal sticks to the standard tournament.

1. Introduction

Tournaments are frequently used by private corporations, e.g. in the form of job-promotion tournaments or to decide on relative performance pay. The tournament organizer – the principal – is interested in the optimal design of the tournament, that is, in the prize structure that offers the best compromise between implemented efforts and corresponding labor costs.

Nalebuff and Stiglitz (1983) introduced the idea of complementing a tournament by a gap as a minimum distance by which the best performing agent must beat the second best to become the tournament winner. Such a gap is always feasible if the principal measures performance on a cardinal scale. Later it was realized that in designing a tournament, the principal should also take into account whether the performance measure is objective or subjective. We follow Prendergast and Topel (1996), among many others, and address the case where the evaluation of agents may involve an element of subjectivity so that performance measures are unverifiable. Such environment typically holds for labor relationships.[1] We analyze under which conditions the introduction of a gap leads to a strict improvement of the standard tournament that solely specifies prizes.

In our paper, we combine contract theory with the theory of contests. We consider a moral-hazard situation in which a risk

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1 "Objective measures of employee performance are rarely available" (Prendergast and Topel, 1996, p. 958).

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neutral principal designs the optimal tournament contract for two risk averse agents with either unlimited or limited liability. The contract has three elements — a winner prize, a loser prize and a gap. As emphasized by Malcomson (1984, 1986), Rosen (1988, p. 85) and Milgrom and Roberts (1992, p. 369), focusing attention on the class of tournament contracts in a situation with unverifiable performance measure is justified by the fact that individual incentive schemes like piece rates or bonuses do not work: if the performance measure is unverifiable, a rational principal will ex post always claim poor performance of the agents to retain the incentive pay and, hence, to save labor costs. This opportunistic behavior can be anticipated by the agents who consequently choose zero efforts.

Tournament contracts, however, do not require verifiable performance because the principal can credibly commit to pay out a certain collective amount of money as tournament prizes and this outpayment is verifiable by a third party. Since the principal must distribute the tournament pay among the agents, there is no reason to misrepresent the agents' performance any longer, which thus restores agents' incentives.

Our results show that under unlimited liability the introduction of a positive gap leads to a better solution of the fundamental trade-off between incentives and insurance, which is inherent in any moral-hazard problem with risk averse agents. This trade-off already exists in the basic model with one agent: the principal should use pay-for-performance and share the income risk with the agent for incentive reasons, but the efficient allocation of risk would require perfect insurance of the risk averse agent by the risk neutral principal. Since such perfect insurance would erase any incentives, the optimal compensation must lead to a compromise between incentives and insurance.

This fundamental logic also applies to tournament contracts. Using a gap yields a partial insurance of the agents when combining it with an optimal prize payment rule for the case that neither contestant has won by the gap. A randomized distribution rule (e.g., tossing a coin) cannot be optimal since the agents are risk averse. Giving each agent the average of the winner and loser prizes is optimal under risk aversion and unlimited liability. If agents are risk averse and, hence, have a concave utility function, an agent's utility from receiving the average pay is larger than the expected utility from receiving the winner and loser prizes each with probability one half in a symmetric tournament equilibrium.

Such partial insurance of agents is beneficial for the principal under unlimited liability. As is known from the basic one-agent moral-hazard model, an agent's participation constraint is always binding given the optimal contract and unlimited liability. The same rationale holds for two agents and the optimal tournament contract which makes the agents just indifferent between receiving their reservation value and participating in the tournament. An increase of the agents' expected utility via partial insurance directly benefits the principal because he can save money by lowering the loser prize without violating the agents' participation constraint. The principal's optimization problem is complicated by the fact that the use of a positive gap is not free of cost. For given tournament prizes, incentives are maximized by a zero gap, so that introducing a gap is detrimental from a pure incentive perspective. However, our results point out that the principal can always adapt his flexible tournament prizes so that the partial insurance by the gap leads to a first-order gain for the principal that dominates the second-order incentive loss.

If agents are protected by limited liability and earn positive rents, the principal will not be interested in partially insuring the agents against income risks any longer as the agents' participation constraints are not binding in the optimum. Since incentives are maximized by a zero gap, the principal prefers to keep to the standard tournament without gap.

As our paper, Eden (2007) analyzes a tournament model that is based on the seminal paper by Lazear and Rosen (1981). She shows that supplementing a standard tournament by a gap will be optimal if the tournament prizes are exogenously given and if prizes need not to be paid out in any case in order to satisfy Malcomson's (1984) self-commitment property. However, if tournament prizes must always sum up to the same constant, the standard tournament contract without gap will be optimal. This result corresponds to our finding under limited liability. Kono and Yagi (2008, p. 124) argue that, in a related model, introducing a positive gap may increase agents' incentives. In our model, introducing a gap decreases incentives, but we show that the loss is outweighed by the gain due to the insurance effect if agents are risk averse and have unlimited liability. Imhof and Kräkel (2013a) analyze how a gap can be used to balance competition under biased performance evaluation, whereas Imhof and Kräkel (2013b) show how a gap can be used to reduce agents' rents. However, both papers assume agents to be risk neutral so that insurance of agents cannot be an issue.

2. The model

We consider a situation where a principal must hire two agents in order to run a business.[2] The principal is risk neutral whereas the two agents are assumed to be risk averse. In particular, let agent i's ($i = 1, 2$) utility from earning income I_i and exerting effort e_i be given by

$$U(I_i, e_i) = u(I_i) - c(e_i) \tag{1}$$

with $u(I_i)$ being monotonically increasing and strictly concave with $u(0) = 0$, and c satisfying $c(0) = c'(0) = 0$ and $c'(e_i), c''(e_i) > 0$ for $e_i > 0$. Hence, we have $U(0, 0) = 0$. Let each agent's reservation utility be $\bar{U} = 0$.

The principal wants to implement a certain effort level at lowest possible cost. For each agent i ($i = 1, 2$), he observes the unverifiable performance signal $x_i(e_i) = h(e_i) + \theta_i$ with $h(0) = 0$ and $h'(e_i) > 0, h''(e_i) \leq 0$. The variables θ_1 and θ_2 denote agents' luck being i.i.d. with density f and cdf F. We assume that $\int_{-\infty}^{\infty} f^2(\theta)\, d\theta < \infty$ to guarantee that $\theta_1 - \theta_2$ has a continuous density g with corresponding cdf G. The principal can neither observe e_i (or $h(e_i)$) nor θ_i so that we have a typical moral-hazard problem.

To induce incentives, the principal uses a tournament that specifies a winner prize w_H, a loser prize $w_L < w_H$ and a gap $\gamma \geq 0$ by which the better performing agent must outperform his opponent to get the winner prize. In other words, agent i will only receive w_H if $x_i(e_i) > x_j(e_j) + \gamma$. In that case, agent j obtains the loser prize w_L. As explained in the introduction, each agent receives $(w_H + w_L)/2$ in case of a tie, i.e., if $|x_1(e_1) - x_2(e_2)| \leq \gamma$. We consider two scenarios: if the agents are not protected by limited liability, there will be no further restriction on the choice of w_H and w_L; if agents are protected by limited liability, we assume that $w_H, w_L \geq 0$ must hold.

3. Solution to the game

First, we solve the tournament game between the two agents. Then we answer the question how the principal should design w_H, w_L and γ to implement a certain effort level at lowest cost.

Agent 1 maximizes

$$\begin{aligned} EU_1(e_1) = {} & u(w_H) \cdot [1 - G(h(e_2) - h(e_1) + \gamma)] \\ & + u(w_L) \cdot G(h(e_2) - h(e_1) - \gamma) \\ & + u\left(\frac{w_H + w_L}{2}\right) \cdot [G(h(e_2) - h(e_1) + \gamma) \\ & - G(h(e_2) - h(e_1) - \gamma)] - c(e_1). \end{aligned}$$

[2] Most of the assumptions follow Lazear and Rosen (1981).