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Reference technology sets, Free Disposal Hulls and productivity decompositions

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HIGHLIGHTS

- Productivity growth is decomposed into explanatory factors.
- Relative unrestrictive regularity conditions are used.
- Specification of the relevant reference technologies is addressed.
- Context is a panel of production units and Free Disposal Hull methods.

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ABSTRACT

Diewert and Fox (2013) proposed decompositions of a Malmquist-type productivity index into explanatory factors, with a focus on extracting technical progress, technical efficiency change and returns to scale components. A major problem with their decompositions is that it may be difficult to determine the appropriate reference technologies. Using relatively unrestrictive regularity conditions, the paper develops a data envelopment type approach for decomposing productivity growth for a panel of production units into explanatory factors based on the Free Disposal Hull methods pioneered by Tulkens and his co-authors.

1. Introduction

Following the definition of Malmquist input, output and productivity indexes by Caves et al. (1982), there has been much interest in the specification and decomposition of productivity growth based on these theoretical indexes; see e.g. Färe et al. (1994) and Grosskopf (2003). In proposing their decompositions of a Malmquist-type productivity index into explanatory factors of technical progress, technical efficiency change and returns to scale components, Diewert and Fox (2013) followed Bjurek (1996) in defining Malmquist indexes for a production unit when the knowledge of the reference best practice technology is available for the two periods under consideration. However, they did not address the problems associated with the determination of the appropriate reference technologies.

In this paper, we assume that a panel data set is available for two periods on K production units that are assumed to be producing the same set of M outputs and using the same set of N inputs and we assume that the *Free Disposal Hulls* (FDHs) generated by these data form the reference technology sets for the two periods under consideration; see e.g. Deprins et al. (1984), Tulkens (1993), Tulkens and Eeckaut (1995a,b) and Daraio and

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Simar (2005) on the FDH approach, and Lovell (2006) for a review. Thus our methodology follows the same path as the better-known data envelopment analysis (DEA) methodologies (see e.g. Charnes and Cooper, 1985) except that the reference technology sets use Free Disposal Hulls instead of Convex Free Disposal Hulls. Russell and Schworm (2009) informatively refer to these types of reference technology sets as *data generated technologies*.

We follow in the footsteps of Tulkens and his co-authors but we also indicate how the various output and input distance functions that form the theoretical Malmquist indexes can be constructed using these reference technology sets so that we can obtain the productivity decompositions suggested by Diewert and Fox (2013).

2. Productivity growth decomposition

Diewert and Fox (2013) derived explanatory factor decompositions of the Bjurek (1996) productivity index, which they defined as follows:

$$\begin{aligned}\Pi_B(x^0,x^1,y^0,y^1) \equiv \{&[d^0(y^1,x^0)/d^0(y^0,x^0)]\\ &\times[D^0(y^0,x^0)/D^0(y^0,x^1)]\\ &\times[d^1(y^1,x^1)/d^1(y^0,x^1)]\\ &\times[D^1(y^1,x^0)/D^1(y^1,x^1)]\}^{1/2},\end{aligned} \tag{1}$$

where using the period t reference technology set S^t, and given a nonnegative, nonzero output vector $y > 0_M$ and a strictly positive input vector $x \gg 0_N$,[1]

$$D^t(y,x) \equiv \max_{\delta>0}\{\delta : (y, x/\delta) \in S^t\} \tag{2}$$

is the period t *input distance function* D^t for periods $t = 0, 1$, and

$$d^t(y,x) \equiv \min_{\delta>0}\{\delta : (y/\delta, x) \in S^t\}, \tag{3}$$

is the period t *output distance function* for periods $t = 0, 1$.

Diewert and Fox (2013) showed that the distance functions in (2) and (3), and hence the productivity index in (1), are well defined under the following relatively unrestrictive regularity conditions on the reference technology set S^t.

P1. S is a nonempty closed subset of the nonnegative orthant in the Euclidean $M + N$ dimensional space.
P2. For every $y \geq 0_M$, there exists an $x \geq 0_N$ such that $(y, x) \in S$.
P3. $(y, x^1) \in S, x^2 \geq x^1$ implies $(y, x^2) \in S$, i.e. free disposability of inputs.
P4. $y > 0_M$ implies that $(y, 0_N) \notin S$.
P5. $x \geq 0_N, (y, x) \in S$ implies $0_M \leq y \leq b(x)1_M$ where 1_M is a vector of ones of dimension M and $b(x) \geq 0$ is a finite nonnegative bound.
P6. $x \gg 0_N$ implies that there exists $y \gg 0_M$ such that $(y, x) \in S$.
P7. $(y^1, x) \in S, 0_M \leq y^0 \leq y^1$ implies $(y^0, x) \in S$, i.e. free disposability of outputs.

The focus here will be on the Diewert and Fox (2013) exact Fisher type decomposition of the Bjurek productivity index using an output (rather than an input) orientation. The Bjurek (1996, 310–311) productivity index Π_B defined by (1) is the geometric mean of the Laspeyres and Paasche type Bjurek indexes where the latter indexes are basically Malmquist (1953) output indexes divided by Malmquist input indexes. The Bjurek productivity index turns out to have the following decomposition:

$$\Pi_B(x^0,x^1,y^0,y^1) = [\varepsilon^1/\varepsilon^0]\tau\rho \tag{4}$$

where x^t and y^t are, respectively, the observed input and output vectors for a production unit for time periods $t = 0, 1$. The period t *technical efficiency* of the unit for period t, ε^t, is defined as:

$$\varepsilon^t \equiv d^t(y^t,x^t) = \min_{\delta}\{\delta : (y^t/\delta, x^t) \in S^t\} \leq 1, \tag{5}$$

so that $[\varepsilon^1/\varepsilon^0]$ measures the efficiency change over the two periods. The Diewert and Fox *technical change* measure, τ, is defined as:

$$\tau \equiv \{[d^0(y^0,x^0)/d^1(y^0,x^0)][d^0(y^1,x^1)/d^1(y^1,x^1)]\}^{1/2} \tag{6}$$

and the Diewert and Fox *returns to scale measure*, ρ, turns out to equal the following expression[2]:

$$\rho = [\varepsilon^0/\varepsilon^1]\tau^{-1}\Pi_B(x^0,x^1,y^0,y^1). \tag{7}$$

A careful examination of the various distance function components of the decomposition shows that it is necessary to be able to calculate the following four *input distances* for a representative production unit that has output and input vectors (y^0, x^0) and (y^1, x^1) for periods 0 and 1: $D^0(y^0, x^0)$, $D^0(y^0, x^1)$, $D^1(y^1, x^1)$ and $D^1(y^1, x^0)$. We also need to compute the following six *output distances*: $d^0(y^0, x^0)$, $d^0(y^1, x^0)$, $d^0(y^1, x^1)$, $d^1(y^1, x^1)$, $d^1(y^0, x^1)$ and $d^1(y^0, x^0)$. We will show how this can be done by using Free Disposal Hulls generated by a panel data set of outputs and inputs covering two periods and K production units, (y^{tk}, x^{tk}) for $t = 0, 1$ and $k = 1, \ldots, K$.

3. Approximations to reference technology sets

To avoid regularity problems, we assume that the input vectors are strictly positive and the output vectors are nonnegative with the first component always being positive; i.e., we assume that:

$$\begin{aligned}&x^{tk} \gg 0_N; \qquad y^{tk} = [y_1^{tk},\ldots,y_M^{tk}] \geq 0_M \quad \text{and}\\ &y_1^{tk} > 0 \quad \text{for } t = 0, 1 \text{ and } k = 1,\ldots,K.\end{aligned} \tag{8}$$

We assume that there are industry best practice technology sets S^0 and S^1 satisfying P1–P7 with

$$(y^{0k}, x^{0k}) \in S^0 \quad \text{and} \quad (y^{1k}, x^{1k}) \in S^1 \quad \text{for } k = 1,\ldots,K. \tag{9}$$

We also assume that there is *no technological regress* in the industry going from period 0 to 1 so that S^0 is a subset of S^1:

$$S^0 \subset S^1. \tag{10}$$

For more on this assumption, see e.g. Diewert (1980, 264) and Diewert (1981, 27–28) in the context of a DEA approach to measuring technical progress, and Tulkens and Eeckaut (1995a,b).

The set of input vectors x that can produce at least the output vector $y \geq 0_M$ using the period 0 technology, $S^0(y)$, is defined as follows:

$$S^0(y) \equiv \{x : (y, x) \in S^0\}. \tag{11}$$

Assumptions (9) imply that $x^{0k} \in S^0(y^{0k})$ for $k = 1,\ldots,K$. Thus by the input free disposability assumption on S^0, the orthant $\{x : x \geq x^{0k}\}$ is a subset of $S^0(y^{0k})$ for $k = 1,\ldots,K$.

Now let $y \geq 0_M$ and define $\alpha^0(y; y^{01}, y^{02},\ldots,y^{0K})$ as the set of indices i such that y^{0i} is equal to or greater than y; i.e.,

$$\alpha^0(y; y^{01}, y^{02},\ldots,y^{0K}) \equiv \{i : y^{0i} \geq y;\ i = 1,2,\ldots,K\}. \tag{12}$$

[1] Notation: $y \geq 0_M$ means each component of the vector y is nonnegative, $y \gg 0_M$ means that each component is strictly positive, and $y > 0_M$ means $y \geq 0_M$ but $y \neq 0_M$.

[2] See Diewert and Fox (2013) for their underlying definition of returns to scale. Their measure is basically a geometric average of output growth divided by input growth for the production unit under consideration but the output and input growth measures hold the technology constant using the period 0 and 1 reference technologies.