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Modelling non-stationary variance in EEG time series by state space GARCH model

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Abstract

We present a new approach to modelling non-stationarity in EEG time series by a generalized state space approach. A given time series can be decomposed into a set of noise-driven processes, each corresponding to a different frequency band. Non-stationarity is modelled by allowing the variances of the driving noises to change with time, depending on the state prediction error within the state space model. The method is illustrated by an application to EEG data recorded during the onset of anaesthesia.

Keywords: State space model; Kalman filter; Frequency decomposition; Autoregressive model; Conditional heteroscedasticity; Non-stationarity; Anaesthesia

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1. Introduction

Brain dynamics can be analyzed by estimating the frequency spectrum of the electroencephalogram (EEG), which can be done by parametric or non-parametric methods; fast Fourier transform (FFT) represents a well-known non-parametric method [1,2], while fitting of autoregressive (AR) models is a prominent example of a parametric method [3,4]. However, in the case of the presence of pronounced

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non-stationarity in the EEG, such as time-dependent changes of the power in different frequency bands, direct application of the FFT to the data would be inappropriate.

Although in this case it is still possible to apply the FFT to a window moving over the data, such approach would have the disadvantage of reduced resolution either in time or in frequency domain; improved resolution in time domain, desirable in order to pick out distinctive temporal characteristics in the data, has to be paid by reduced resolution in frequency domain, and vice versa.

In contrast, parametric spectral estimation by AR models offers various advantages over the FFT, since it represents a more general and flexible framework for parsimonious dynamical modelling of time series data, which can be readily employed for purposes such as prediction, classification or causality analysis of time series [5]; in the case of non-stationarity, parametric spectral estimation may also be applied to a moving window [6], but as we will show in this paper, there is an alternative approach for this situation which avoids the introduction of a moving window.

We will model the EEG by a linear autoregressive model in a state space framework, which is suitable for describing the simultaneous presence of several major frequency bands; the non-stationarity will be added by employing the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, as introduced by Engle [7] for the modelling of time-dependent variance, and generalized by Bollerslev [8], but in contrast to the usual usage of the GARCH model we will apply it within state space.

The concept of employing the GARCH model within a state space has been introduced by Galka et al. [9] in a study on the estimation of inverse solutions from EEG time series; for simulated data they obtained improved reconstruction of true states by this technique.

It is a property of the GARCH model that non-stationarities can be detected with very good temporal resolution [8]; therefore we propose it as an appropriate tool for the modelling of transients and rapid changes of spectral properties, as they are commonly observed in EEG time series. As an example we will discuss in this paper the case of a clinical EEG time series displaying the transition from awake conscious state to anaesthesia.

2. An example serving as motivation

The EEG time series which we will study in this paper was retained from a recent study of John et al. [10] and John [11] who have studied the change of spectral content of clinical EEG accompanying the loss and subsequent recovery of consciousness due to initiation and termination of anaesthesia during surgery. The data was measured at 19 electrodes fixed to the scalp according to the international 10/20 System. The detailed experimental procedures have been described in John et al. [10] and Prichep et al. [12].

Based on techniques from descriptive statistics, including FFT spectrum estimation and computation of mean z-scores, John et al. [10] found that an increase in absolute power of the low frequency band occurs when patients lose consciousness. In this paper we will study the same topic and confirm their result by a parametric approach.

We select from their data a segment of 2048 samples from the T4 electrode (versus average reference), sampled at 100 Hz, such that the segment extends over about 20 s. This data set covers the transition from awake conscious state to anaesthesia. The data is shown in Fig. 1.

At time 0 s, induction of anaesthesia begins. At about 10 s, loss of consciousness occurs. It can be seen in the figure that starting from this time the amplitude of low-frequency activity in the EEG displays a pronounced increase.