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A dynamic general equilibrium model of pollution abatement under learning by doing

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HIGHLIGHTS

- Extend the model of Beladi et al. (2013) with learning by doing.
- Investigate the dynamic general equilibrium model of pollution abatement.
- Analyze the steady-state equilibrium properties of emission permits and pollution treatment.
- Derive the steady-state optimal levels of emission permits and pollution treatment.

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ABSTRACT

In 2013, Beladi et al. constructed a dynamic general equilibrium model of pollution, and characterized a steady-state equilibrium. In this paper, we extend Beladi et al.'s model to an even more general model in which the pollution abatement costs under learning by doing are taken into account. In our model, the instantaneous abatement costs depend on both the rate of abatement and the experience of using a technology. Our objective is to apply optimal control theory to investigate the dynamic general equilibrium model of pollution abatement, and derive the steady-state equilibrium properties and optimal levels of emission permits and pollution treatment.

1. Introduction

In a recent work, Beladi et al. (2013) constructed a dynamic general equilibrium model of pollution and characterized a steady state equilibrium. A noted feature of the authors' paper is taking a holistic view of environmental policies where the government chooses an optimal emission cap, issues permits and takes pollution abatement activity to clean up the environment. As the government sells emission permits, in a sense it can be viewed as imposing taxes on emission. So the authors' view takes all three major facets of environmental policies into consideration together.

In this paper, following the analytical framework of Beladi et al. (2013), we present a dynamic general equilibrium model of pollution abatement under learning by doing. Furthermore, our analysis is in the spirit of the study by Bramoullé and Olson (2005)—the first to obtain the analytical expression for the experience using pollution abatement technology—as well as more recent works by Argotte and Epple (1990) and Greaker and Rosendahl (2008). Another related paper is by Goulder and Mathai (2000), in which the authors investigate the problem of abatement-cost function. Goulder et al.'s abatement-cost function differs from ours, however, in that it focuses on learning that improves abatement technologies and thus reduces abatement costs. In our model, the experience using pollution abatement technology is measured by the cumulative abatement from time 0 to t. This assumption is consistent with empirical studies of learning by doing, in which experience is generally measured as cumulative production. The instantaneous abatement costs depend on both the rate of abatement and the experience of using a technology.

This paper is organized as follows. In the next section we present our dynamic general equilibrium model of pollution abatement. Section 3 derives our steady-state equilibrium properties and optimal levels of emission permits and pollution treatment. We summarize the results in Section 4.

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2. The basic model

We start out with the simple but widely used model in which an economy produces two goods, 1 and 2. Let Y_i, E_i and L_i be the production level, emission level and labor usage of sector i, respectively, $i = 1, 2$. According to Beladi et al. (2013), the production level is given by the following Cobb–Douglas production function:

$$Y_i = E_i^{\alpha_i} L_i^{\beta_i} \tag{1}$$

where α_i and β_i are positive constants. Here we assume that both sectors exhibit constant returns to scale, i.e., $\alpha_i + \beta_i = 1$, and that $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$ implying that sector 1 is labor intensive and sector 2 is emission intensive.

Following Beladi et al. (2013), the consumers' preference or utility function can be represented as

$$U = C_1^{\theta_1} C_2^{\theta_2} X^{-\gamma} \tag{2}$$

where C_i is the consumption of goods $i, i = 1, 2$; X is the accumulated stock of pollution; $\gamma \in (0, 1), \theta_i \in (0, 1),\ i = 1, 2$. In order to ensure the concavity of the utility function, we assume that $\theta_1 + \theta_2 = 1$.

Let $A(t)$ be the pollution abatement level at time t. According to Bramoullé and Olson (2005), experience of using pollution abatement technology is measured by the cumulative abatement from time 0 to t and is given by the following form:

$$Z(t) = Z_0 + \mu \lim_{t\to\infty} \int_0^t A(s)ds \tag{3}$$

where Z_0 denotes the initial level of experience using pollution abatement technology, μ is the positive constant. This assumption is consistent with empirical studies of learning by doing, in which experience is generally measured as cumulative production (Argotte and Epple, 1990).

Following Bramoullé and Olson (2005), the instantaneous abatement cost $C(A(t), Z(t))$ depends on both the rate of abatement and the experience of using a pollution abatement technology, and the instantaneous abatement cost $C(A(t), Z(t))$ has the following properties: (i) $C(A(t), Z(t))$ is twice differentiable; (ii) $C(A(t), Z(t))$ is increasing in A and decreasing in Z; (iii) $C(A(t), Z(t))$ is convex in $(A(t), Z(t))$; (iv) $C(0, Z(t)) = 0$. The abatement cost is increasing and convex in abatement and decreasing and convex in experience. Learning reduces the abatement cost at a decreasing rate and the gains from experience are higher when experience is low.

Following Bramoullé and Olson (2005) and Goulder and Mathai (2000), the instantaneous pollution abatement cost $C(A(t), Z(t))$ function is assumed to take the form:

$$C(A(t), Z(t)) = b_1 A(t)^{(\sigma-\gamma)} - b_2(Z(t) - Z_0) \tag{4}$$

where b_1 and b_2 are constants; $\sigma = \theta_1\alpha_1 + \theta_2\alpha_2$, and we assume that $\gamma \le \sigma$.

The dynamics of pollution stock $\dot{X}(t)$ is prescribed by the ordinary differential equation

$$\dot{X}(t) = E(t) - A(t) - \eta X(t) \tag{5}$$

where $\eta > 0$ is a constant decay rate of pollution.

Let ω and τ be the economy wide wage rate and the pollution permit price, P_i be the price of goods $i, i = 1, 2$, and we assume that both goods markets are competitive. Then, we can get the following expressions:

$$E_i = \frac{P_i\alpha_i Y_i}{\tau}, \quad i = 1, 2 \tag{6}$$

$$L_i = \frac{P_i\beta_i Y_i}{\omega}, \quad i = 1, 2. \tag{7}$$

We normalize the initial goods prices to unity throughout the paper. The labor and emission permits market clearing conditions are:

$$E_1 + E_2 = E \tag{8}$$

$$L_1 + L_2 = L \tag{9}$$

where L is the constant stock of labor, and E is the total amount of pollution permits to be determined by the government.

Now, we consider the consumers and firms' problems. The consumers face the following budget constraint

$$P_1C_1 + P_2C_2 = \omega L. \tag{10}$$

The consumers' maximization problem leads to the demand functions

$$C_i = \frac{\theta_i \omega L}{P_i}, \quad i = 1, 2. \tag{11}$$

Similarly, the firms' demand for labor and permits can be derived as $E_i = \frac{P_i\alpha_i Y_i}{\tau}$, $L_i = \frac{P_i\beta_i Y_i}{\omega}$, $i = 1, 2$. Using market clearing conditions $C_i = Y_i$, $i = 1, 2$, we obtain that $E_1/E_2 = \theta_1\alpha_1/\theta_2\alpha_2$, and $L_1/L_2 = \theta_1\beta_1/\theta_2\beta_2$. Using these relationships between equilibrium input usage as well as Eqs. (10) and (11), one obtains:

$$E_i = \frac{\theta_i\alpha_i}{\theta_1\alpha_1 + \theta_2\alpha_2} E \tag{12}$$

$$L_i = \frac{\beta_i\theta_i}{\beta_1\theta_1 + \beta_2\theta_2} L. \tag{13}$$

Substituting Eqs. (12) and (13) into (1), using the market clearing conditions once again, and substituting the resulting equilibrium consumption levels in Eq. (2), we obtain the following money-metric indirect utility function:

$$V = \Upsilon(L) E(t)^\sigma X(t)^{-\gamma} \tag{14}$$

where $\Upsilon(L) = \Lambda \left(\frac{\psi}{1+\psi}\right)^{\beta_1\theta_1+\beta_2\theta_2} L^{\beta_1\theta_1+\beta_2\theta_2}$, $\psi = \frac{\beta_1\theta_1+\beta_2\theta_2}{\theta_1\alpha_1+\theta_2\alpha_2}$,

$$\Lambda = \left(\frac{\theta_1\alpha_1}{\theta_1\alpha_1+\theta_2\alpha_2}\right)^{\alpha_1\theta_1} \left(\frac{\beta_1\theta_1}{\beta_1\theta_1+\beta_2\theta_2}\right)^{\beta_1\theta_1} \times \left(\frac{\theta_2\alpha_2}{\theta_1\alpha_1+\theta_2\alpha_2}\right)^{\alpha_2\theta_2} \left(\frac{\beta_2\theta_2}{\beta_1\theta_1+\beta_2\theta_2}\right)^{\beta_2\theta_2}.$$

According to Jehle and Reny (2011), a consumer's indirect utility function gives the consumer's maximal utility when faced with a price level and an amount of income. It represents the consumer's preferences over market conditions. The indirect utility function for the consumer is analogous to the profit function for the firm. So we can measure the indirect utility function by using money-metric. Eq. (14) is a money-metric indirect utility function which gives the consumer's maximal utility when faced with price level P_i of the consumption goods C_i and an amount of income $\omega L,\ i = 1, 2$.

Further, according to Rubio and Casino (2002), the problem for the government consists in maximizing with regard to controls E and A the expected value of the following functional:

$$\max_{E,A} \int_0^\infty e^{-rt}[\Upsilon(L)E(t)^\sigma X(t)^{-\gamma} - (b_1A(t)^{(\sigma-\gamma)} - b_2(Z(t) - Z_0))]dt. \tag{15}$$

In the next section, we apply optimal control theory to find the optimal levels of abatement and pollution permits such that the discounted stream of welfare is maximized.