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Long- versus medium-run identification in fractionally integrated VAR models

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HIGHLIGHTS

- Fractional VAR with integration orders < 1: long-run identification lacks interpretation.
- Consider medium-run information to obtain identified shocks.
- Three such approaches are presented.
- Asymptotic equivalence to long-run restriction shown for large horizons.

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ABSTRACT

We state that long-run restrictions that identify structural shocks in VAR models with unit roots lose their original interpretation if the fractional integration order of the affected variable is below one. For such fractionally integrated models we consider a medium-run approach that employs restrictions on variance contributions over finite horizons. We show for alternative identification schemes that letting the horizon tend to infinity is equivalent to imposing the restriction of Blanchard and Quah (1989) introduced for the unit-root case.

1. Introduction

Correct specification of integration orders is essential for valid inference in structural vector autoregressive (SVAR[1]) models, in particular, if identification of the structural shocks is related to their long-run effects. Therefore, the literature considered fractional time series models where the orders of integration may take on real (instead of integer) values and are estimated along with the other model parameters.

Recent results suggest that macroeconomic variables such as GDP or inflation may have integration orders smaller than one; see, e.g., Caporale and Gil-Alana (2013) or Gil-Alana (2011). This means that in multivariate fractional integration models, no shock could have an infinitely long-living effect on these variables, regardless of structural restrictions.

As a remedy we suggest the use of medium-run constraints for identification, which was considered in standard SVARs by Uhlig (2004) and Francis et al. (forthcoming) as an alternative to long-run restrictions. We propose several approaches which constrain the variance contribution of selected shocks over a prespecified range of periods. For these finite-horizon criteria we show that by letting the number of periods tend to infinity they become formally identical to the computationally straightforward Blanchard and Quah (1989) condition. We thus provide an economic interpretation of the latter and justify its use in a fractional context.

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[1] Abbreviations used in the text: (S)VAR: (Structural) vector autoregression, FIVAR: Fractionally integrated vector autoregression, LRR: Long-run restriction, LRRS: Long-run restricted shock, LRUS: Long-run unrestricted shock, ARFIMA: Autoregressive fractionally integrated moving average.

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2. Fractional SVARs and identification

2.1. The model

In order to avoid the restriction of integer integration orders for structural VAR analysis, fractionally integrated vector autoregressive (FIVAR) models have been used; see, e.g., Caporale and Gil-Alana (2011), Gil-Alana and Moreno (2009) or Lovcha (2009). Tschernig et al. (2013a) introduced additional flexibility for the short-run dynamics by a fractional lag operator. The subsequent analysis will be based on the popular FIVAR model, noting that the results straightforwardly carry over to the more flexible model of Tschernig et al. (2013a) as well. We assume that the bivariate time series $\boldsymbol{x}_t = (x_{1t},\ x_{2t})'$ is generated by

$$\boldsymbol{A}(L)\boldsymbol{\Delta}(L;\boldsymbol{d})\boldsymbol{x}_t = \boldsymbol{B}\boldsymbol{\varepsilon}_t, \quad t = 1, 2, \dots, \tag{1}$$

where L is the lag or backshift operator ($L\boldsymbol{x}_t = \boldsymbol{x}_{t-1}$) and $\boldsymbol{\Delta}(L;\boldsymbol{d}) := \mathrm{diag}(\Delta^{d_1}, \Delta^{d_2})$ holds the fractional difference operators $\Delta^{d_j} = (1-L)^{d_j}$ of real orders d_1 and d_2 (see, e.g., Baillie, 1996). Starting values are set to zero, i.e., $\boldsymbol{x}_t = \boldsymbol{0}$ for $t < 1$ although this assumption can be relaxed along the lines of Johansen (2008).

Analogous to standard non-cointegrated VAR models with unit roots, the fractionally differenced series $(\Delta^{d_1}x_{1,t}\ \ \Delta^{d_2}x_{2,t})'$ follows a stable VAR model with all roots of $\boldsymbol{A}(L) = \boldsymbol{I} - \boldsymbol{A}_1L - \cdots - \boldsymbol{A}_pL^p$ outside the unit circle. In our structural setup, $\boldsymbol{\varepsilon}_t \sim IID(\boldsymbol{0};\boldsymbol{I})$ is the vector of economic shocks and the impact matrix $\boldsymbol{B}$ holds their contemporaneous effects.

2.2. Long-run and finite-horizon identification schemes

Identification restrictions are needed to uniquely recover the elements of the impact matrix $\boldsymbol{B}$ from the reduced-form error covariance matrix $\boldsymbol{\Omega} = \mathrm{Var}(\boldsymbol{u}_t) = \boldsymbol{B}\boldsymbol{B}'$, where $\boldsymbol{u}_t = \boldsymbol{B}\boldsymbol{\varepsilon}_t$ is the reduced-form disturbance term. To this end, Blanchard and Quah (1989) introduced the concept of long-run restrictions which exclude an infinitely long-lasting impact of selected shocks on a specific variable. In a setup with $d_1 = d_2 = 1$, denote $\boldsymbol{\Xi}(z) := \sum_{j=0}^{\infty}\boldsymbol{\Xi}_j z^j = \boldsymbol{A}(z)^{-1}\boldsymbol{B}$. The effect of a shock in $\boldsymbol{\varepsilon}_t$ on $\boldsymbol{x}_{t+h}$ in the distant future, $h \to \infty$, is given by $\boldsymbol{\Xi}(1)$. Identification of $\boldsymbol{B}$ can be obtained by constraining the permanent effect of, say, $\varepsilon_{2,t}$ on the first variable $x_{1,t}$ using the long-run restriction (LRR)

$$\boldsymbol{\Xi}(1) = \boldsymbol{A}(1)^{-1}\boldsymbol{B} = \begin{pmatrix} \xi_{11}(1) & 0 \\ \xi_{21}(1) & \xi_{22}(1) \end{pmatrix}. \tag{2}$$

We keep this ordering of shocks and refer to $\varepsilon_{1,t}$ as the long-run unrestricted shock (LRUS), while $\varepsilon_{2,t}$ is called the long-run restricted shock (LRRS). Below we will show that in fractional models the LRR (2) loses its original interpretation but features the meaning of a medium-run restriction in the limiting case.

Let $\boldsymbol{x}_t = \boldsymbol{\Theta}_0\boldsymbol{\varepsilon}_t + \boldsymbol{\Theta}_1\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\Theta}_2\boldsymbol{\varepsilon}_{t-2} + \cdots$ denote the vector moving average representation of model (1), and denote by $\theta_{ij,h}$ the ijth element of $\boldsymbol{\Theta}_h$, i.e. the impulse responses of the ith variable to the jth structural shock at horizon h.[2] Shocks to $x_{1,t}$ can have ever lasting effects on future realizations of this variable only if $d_1 \geq 1$. Formally, as established by Kokoszka and Taqqu (1995), the impulse responses generally evolve at a rate of order h^{d_1-1} and thus converge to zero at a hyperbolic rate if $d_1 < 1$. The impact of *any* shock to $x_{1,t}$ vanishes with an increasing horizon so that no long-run effect in the terminology of Blanchard and Quah (1989) exists. The economic interpretation of LRR (2) is no longer obvious in this context.

To clarify this interpretation we apply an approach focusing on specified finite horizons and show how it can approximate the long-term behavior. To quantify the influence of the structural shocks for a given horizon, note that the forecast error of $x_{i,t+h},\ i \in \{1, 2\}$, based on known coefficients and information up to period t is given by $\sum_{s=0}^{h-1}\sum_{j=1}^{2}\theta_{ij,s}\varepsilon_{j,t+h-s}$. Consider the forecast error variance

$$\begin{aligned}\mathrm{Var}_t(x_{i,t+h}) &= \sum_{s=0}^{h-1}\left(\theta_{i1,s}^2 + \theta_{i2,s}^2\right) \\ &= \sum_{s=0}^{h-1}\theta_{i1,s}^2 + \sum_{j=0}^{h-1}\theta_{i2,s}^2, \quad i = 1, 2,\end{aligned} \tag{3}$$

which can be decomposed into one variance component due to the LRUS, $\varepsilon_{1,t}$, and one due to the LRRS, $\varepsilon_{2,t}$. Thus, the share of the h-step forecast variance of variable i due to $\varepsilon_{j,t}$ is given by

$$\omega_{ij,h} = \frac{\sum_{s=0}^{h-1}\theta_{ij,s}^2}{\mathrm{Var}_t(x_{i,t+h})}. \tag{4}$$

In order to require a small impact of the LRRS on the behavior of the first variable h periods ahead, we consider three identification schemes which draw on restricting these variance shares or a variant thereof. We first choose an identification procedure that directly minimizes the forecast error variance share of the LRRS, i.e. FIN1

$$\min_{\boldsymbol{B}} \omega_{12,h} \quad \text{s.t. } \boldsymbol{B}\boldsymbol{B}' = \boldsymbol{\Omega}. \tag{5}$$

Since minimizing the contribution of the restricted shock amounts to maximizing the share of the unrestricted one, in our bivariate model this is identical to the constraint brought forward by Francis et al. (forthcoming).

While economic theory hardly gives any guidance regarding an appropriate value of h, one may instead have an interval of horizons in mind which will be considered relevant. Then it would be reasonable to focus on a range $h \in [l; u]$, over which the LRRS should have minimal impact. Using the average forecast error variance contribution (4) for identification yields FIN2

$$\min_{\boldsymbol{B}} \frac{1}{u-l+1}\sum_{h=l}^{u}\omega_{12,h} \quad \text{s.t. } \boldsymbol{B}\boldsymbol{B}' = \boldsymbol{\Omega}. \tag{6}$$

If a shock $\varepsilon_{2,t}$ has a large effect on $x_{1,t}$ over the first few periods, this is also reflected by the longer-term forecast error variance since short-horizon impulse responses enter FIN1 (5) and FIN2 (6) through the sum in (3). The interpretation of the LRRS as having a restricted effect over longer horizons may suffer from this property. In order to avoid this problem we modify FIN1 and obtain FIN3

$$\min_{\boldsymbol{B}} \frac{\sum_{i=l}^{h}\theta_{12,i}^2}{\mathrm{Var}_t(x_{1,t+h})} \quad \text{s.t. } \boldsymbol{B}\boldsymbol{B}' = \boldsymbol{\Omega}, \tag{7}$$

where now the variance share of exclusively the successive $h-l$ shocks, $\varepsilon_{2,t+1}, \dots, \varepsilon_{2,t+h-l}$, contributing to a $x_{1,t+h}$ is minimized. The restriction proposed by Uhlig (2004) is obtained as a special case by setting $l = h$ for FIN3. The computation of all three finite horizon restrictions is described in the Appendix.

3. Relation between long-run and finite-horizon restrictions

3.1. The long and the medium run in fractional models

Without the typical interpretation, but still referred to as the LRR in the following, restriction (2) can be likewise imposed in the

[2] Chung (2001) discusses computation of impulse responses and their properties in the vector ARFIMA model while Do et al. (2013) introduce conceptually different generalized impulse responses in our FIVAR setup.