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Growth and financial liberalization under capital collateral constraints: The striking case of the stochastic AK model with CARA preferences

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HIGHLIGHTS

- We consider a small-open, collateral-constrained AK economy.
- CARA preferences and uncertainty on capital inflows generates long-term growth.
- Long-term growth is entirely driven by precautionary savings.
- Growth rate is increasing in the risk magnitude and in the risk aversion parameters.
- More financially integrated economies experience lower consumption growth volatility.

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ABSTRACT

We consider a small-open, collateral-constrained AK economy. We show that the combination of CARA preferences and uncertainty on capital inflows generates long-term growth while the deterministic counterpart does not: long-term growth is entirely driven by precautionary savings, and the asymptotic growth rate of the expected capital stock is increasing in both the risk magnitude and the Arrow–Pratt absolute risk aversion parameters.

1. Introduction

Admittedly, a strong argument in favor of financial liberalization is international risk-sharing. As outlined by Obstfeld (1994), portfolio diversification thanks to open international financial assets markets pave the way to significant welfare gains. Obviously, the argument is particularly strong in the absence of imperfections in international financial markets. Recently, Boucekkine et al. (2012) show that financial liberalization is not always welfare-improving when the economies are subject to capital collateral constraints; for financial openness to be welfare-increasing, the corresponding autarkic growth rates should be large enough, which is somehow consistent with the empirical literature (see for example, Kose et al., 2011). Boucekkine et al. (2012) use a deterministic AK model, this note examines a stochastic extension of the Boucekkine et al.'s model. We do not specifically examine Obstfeld's diversification argument but a simpler stochastic extension where uncertainty lies on the magnitude of international financial flows. We more specifically examine the implications of this type of uncertainty for growth.

It is known since Weil (1990) that the presence of risk generates conflicting intertemporal substitution and intertemporal

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income effects, the total outcome depending on risk aversion. An earlier application to stochastic AK models is due to Steger (2005). However, the model considered by Steger is a closed economy and the source of uncertainty is total factor productivity. In this note, we study a small open economy subject to capital collateral constraints, and uncertainty lies on the size of international capital inflows. In particular, the latter modeling gives rise to a square-root Brownian process driving capital accumulation in contrast to Steger's setting which uses a geometric one. In order to get full analytical results with the type of process considered, we resort to constant absolute risk aversion (CARA) utility functions. Of course, CARA functions have some well known specific implications due to the fact that the implied risk premium is wealth-independent. This said, the full resolution of the model with CARA functions shows striking results on the role of uncertainty in the context of our capital collateral constrained open economy. First of all, in the deterministic counterpart, capital grows linearly from $t = 0$, consumption being an affine function of time. This is not surprising at all since CARA functions display intertemporal elasticities of substitution strictly decreasing in consumption. Despite constant returns to capital, the incentives to invest drop as consumption rises, which kills exponential growth. Second, when uncertainty on financial inflows is introduced, the asymptotic growth rates of expected capital and consumption turn out to be strictly increasing functions in the risk magnitude and in the Arrow–Pratt absolute risk aversion parameters. Of course, this striking set of results *entirely* relies on the CARA specification. Nonetheless we believe that it examplifies the working of risk-induced intertemporal substitution effects under imperfect international capital markets, that is the joint role of the latter and precautionary savings. In particular, the model predicts that economies that are more financially liberalized experience lower volatility of consumption growth relative to output growth volatility.[1]

The paper is organized as follows. Section 2 describes the model. Section 3 solves the model and gives the main results. Section 4 concludes.

2. The model

The economy considered is the one described in Boucekkine and Pintus (2012). It is a small open economy endowed with an AK production technology: $Y = A\,K$, $A > 0$. The output good is tradeable, and we assume that the capital input is not (capital immobility). The world interest rate is $r > 0$, and the level of net foreign debt is denoted D. We shall place ourselves in the case of net debtors, so $D \geq 0$. Finally, international borrowing is subject to capital collateral constraints in the spirit of Cohen and Sachs (1986): we assume that at each time $t \geq 0$ we have

$$D(t) = \lambda K(t) \tag{1}$$

for some $\lambda \in [0, 1)$. λ is the credit multiplier, it is a measure of financial markets' imperfection: the larger λ, the lower these imperfections. Boucekkine and Pintus (2012) also study the case of no-commitment, that is when borrowing depends on realized investment (typically past investment). The mathematical treatment required in the latter case is quite heavy (this amounts to the optimal control of functional differential equations). Since we add stochastics, we prefer to build on the more standard deterministic counterpart, that is the one relying on investment commitment.

We now introduce the stochastic structure. Consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and a real standard Brownian motion $W\colon [0, +\infty) \times \Omega \to \mathbb{R}$. Denote by $\mathcal{F}_t$ the filtration generated by W. The state equations describing the evolution of our economy are given by the following system

$$\begin{cases} \mathrm{d}K(t) = (-\delta K(t) + I(t))\ \mathrm{d}t \\ \mathrm{d}D(t) = (rD(t) - AK(t) + I(t) + C(t))\ \mathrm{d}t \\ \qquad - \sqrt{\gamma D(t)}\,\mathrm{d}W(t) \\ K(0) = K_0 > 0, \quad D(0) = D_0 \geq 0, \end{cases} \tag{2}$$

with $\gamma \geq 0$ and $\delta \geq 0$ is the capital depreciation rate (the sign in front of the term $\sqrt{\gamma D(t)}\,\mathrm{d}W(t)$ is irrelevant, we take the minus sign to have the positive sign in our main state equation given here below). The two equations are standard. Notice that uncertainty only affects the law of motion of foreign debt: in contrast to Steger (2005) for example, uncertainty is not technological but totally related to the working of international financial markets, which expand or shrink randomly. Last but not least, it is important to single out the role of parameter γ. This parameter is a quite straightforward measurement of the magnitude of the random financial inflows: the larger it is, the larger this magnitude is likely to be. Notice that when $\gamma = 0$, the model degenerates into the deterministic counterpart.

Using the collateral constraint specification (1), one can reduce the number of state equation to a single (stochastic) one in the capital stock:

$$\begin{cases} \mathrm{d}K(t) = \left(\dfrac{A-\delta-r\lambda}{1-\lambda}K(t) - \dfrac{1}{1-\lambda}C(t)\right)\mathrm{d}t \\ \qquad + \dfrac{\sqrt{\gamma\lambda}\sqrt{K(t)}}{1-\lambda}\,\mathrm{d}W(t) \\ K(0) = K_0 > 0. \end{cases} \tag{3}$$

The stochastic process driving capital accumulation is therefore a non-geometric square-root process. As argued in the Introduction, we set CARA preferences in order to get closed-form solutions to the associated stochastic optimal growth model. More precisely, given two positive constants θ and η we consider the functional

$$J(C(\cdot)) := \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(-\theta e^{-\eta C(t)}\right) dt\right] \tag{4}$$

to be maximized varying the control $C(\cdot)$ under the state Eq. (3). One can directly see that the Arrow–Pratt absolute risk aversion associated to the considered CARA utility function is given by $\eta > 0$.

Define the set of the admissible controls as follows:

$$\mathcal{U}_{K_0} := \Big\{ C(\cdot)\colon [0, +\infty) \times \Omega \to \mathbb{R} : \; C(\cdot) \text{ is } \mathcal{F}^t - \text{progressively measurable and } K(\cdot) \text{ remains positive} \Big\}.$$

Denote with

$$V(K_0) = \sup_{C(\cdot)\in\mathcal{U}_{K_0}} J(C(\cdot)) \tag{5}$$

the value function of the problem. In the next section, we present the main outcomes of the problem, we focus on the relationship between growth and uncertainty, or in other words, between growth and risk-taking.

3. Main results

The first theorem stated just below characterizes the optimal solution to the stochastic optimal control problem by identifying the corresponding value function in closed-form.[2]

[1] Our solution method applies to any economy with CARA preferences subject to square-root Brownian processes on capital. It would definitely work on a closed economy *à la* Steger with the latter preferences and uncertainty on TFP generating a square-root process on capital. The application to the collateral-constrained small open economy is more interesting as it allows to get further results on the implications of international financial integration.

[2] A technical Appendix including the complete proofs of all the results stated is available upon request.