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Ambiguity and perceived coordination in a global game

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HIGHLIGHTS

- Ambiguity is introduced in a simple 2 × 2 global game.
- Larger ambiguity is shown to reduce the amount of coordination each player perceives.
- This is a new channel of the effect of ambiguity in global games.
- Small uncertainty with ambiguity tends to select the Pareto dominated equilibrium.
- Implications for global game models of financial crises are drawn.

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ABSTRACT

In a global game, larger ambiguity is shown to decrease the amount of coordination each player perceives. Consequently, small uncertainty tends to select the Pareto dominated equilibrium of the game without uncertainty. Implications for models of financial crises are drawn.

1. Introduction

The analysis of global games generally uses an expected utility approach to uncertainty. However, as initially underlined by Ellsberg (1961), decision makers seem to exhibit an "aversion to ambiguity": they prefer a situation with known probabilities to a situation with unknown probabilities. Thus, in the last two or three decades, non-Bayesian approaches to uncertainty have been developed in order to take into account ambiguity aversion in such ambiguous situations.[1]

In the present paper, we will use such an approach to uncertainty in a simple 2 × 2 global game. We will underline a new channel through which ambiguity can affect the equilibrium. An increase in ambiguity will be shown to reduce the amount of coordination that each player perceives. As a consequence, when uncertainty becomes small, this will tend to select the Pareto dominated equilibrium of the game without uncertainty, rather than the risk-dominant equilibrium (as in Carlsson and van Damme, 1993). We will also consider the implications of the analysis for global game models of financial crises.[2]

Section 2 presents the model. Section 3 gives the equilibrium of the game. Section 4 studies the effect of uncertainty on the equilibrium. Section 5 considers the equilibrium selection issue. Section 6 draws some implications for models of financial crises. Section 7 concludes.

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[1] Two classical references are Gilboa and Schmeidler (1989) and Schmeidler (1989). For some economic applications, see for example Mukerji and Tallon (2004).

[2] Global games have been applied to currency attacks (Morris and Shin, 1998); to the situation where creditors have to decide to roll over their loans (Morris and Shin, 2004); and to the issue of bank runs (Goldstein and Pauzner, 2005 and Rochet and Vives, 2004).

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2. Model

We consider a symmetric game with two players and two actions, given in Morris and Shin (2003) (which is itself taken from the introductory example of Carlsson and van Damme, 1993). The payoff matrix is

	I	N
I	θ, θ	$\theta - 1, 0$
N	$0, \theta - 1$	$0, 0$

Each player has to decide whether to invest in a project or not. If the player does not invest (action N), the payoff is equal to 0. If the player invests (action I), the payoff depends on some exogenous parameter θ (the strength of "fundamentals") and is greater when both players invest (coordination is beneficial).

If the variable θ were common knowledge, then, in the case $\theta < 0$, action N would be a dominant strategy, while in the case $\theta > 1$, action I would be a dominant strategy. In the case $0 < \theta < 1$, both (I, I) and (N, N) would be the strict Nash equilibria, where (I, I) Pareto dominates (N, N).

However, as in the global game literature, we assume that each player i only receives an imperfect signal x_i on θ, given by

$$x_i = \theta + \xi_i. \quad (1)$$

We will introduce some ambiguity on the mean μ_i of the distribution of ξ_i.[3] We have

$$x_i = \theta + \mu_i + \varepsilon_i. \quad (2)$$

The variables $\varepsilon_1, \varepsilon_2$, and θ are mutually independent. As in Morris and Shin (2003), we assume that θ has a diffuse prior. Let $\mathcal{N}(m, \sigma)$ be the normal distribution of mean m and standard deviation σ. Each ε_i is assumed to follow $\mathcal{N}(0, \sigma)$, with $\sigma > 0$.[4]

Such a signal may be seen as corresponding to a situation where each player receives a signal from an "expert" who, from the available information, gets an opinion which may be biased. This leads to an unknown and ambiguous bias in the signal the player receives. Each player will be assumed to have his/her own expert, who is different from the expert of the other player.[5] As each expert may have his/her own specific bias in interpreting the available information, this implies that we may have $\mu_1 \neq \mu_2$. As we will see, allowing the biases of the signals to be different for the two players will play a crucial role in the analysis.

Each player is ambiguity averse and has a maxmin criterion of expected utility,[6] where each μ_i belongs to the interval $[-\eta, \eta]$, where η represents the amount of ambiguity. The set of possible values for the biases $M \equiv (\mu_1, \mu_2)$ is $\mathcal{M} = [-\eta, \eta] \times [-\eta, \eta]$.

3. Equilibrium

As it is usually done in the global game literature, we will look for equilibria in "switching strategies", where player i chooses action I if $x_i > k$, and chooses action N if $x_i \leq k$. Let $\pi_M(k, x_i, a_i)$ be the expected utility of player i, conditional on having received the signal x_i, when this player takes action a_i and the other player j follows the switching strategy with switching point k, and when M is given. From the payoff matrix, we get $\pi_M(k, x_i, N) = 0$ and

$$\pi_M(k, x_i, I) = E(\theta \mid x_i, M) - \Pr\left(x_j \leq k \mid x_i, M\right). \quad (3)$$

From (2) and our assumptions, the conditional distribution of θ held by player i is $\mathcal{N}(x_i - \mu_i, \sigma)$, and the conditional distribution of $(\varepsilon_1, \varepsilon_2)$ is the same as the unconditional distribution. From (2), we have $\Pr\left(x_j \leq k \mid x_i, M\right) = \Pr\left(\varepsilon_j - \varepsilon_i \leq k - x_i + \mu_i - \mu_j\right)$. As $\left(\varepsilon_j - \varepsilon_i\right)/\sqrt{2}\sigma$ follows $\mathcal{N}(0, 1)$, from (3) we get

$$\pi_M(k, x_i, I) = x_i - \mu_i - \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma} + \frac{\mu_i - \mu_j}{\sqrt{2}\sigma}\right) \quad (4)$$

where $\Phi(\cdot)$ is the cumulative distribution function of $\mathcal{N}(0, 1)$. Player i chooses a_i which maximizes $\min_{M \in \mathcal{M}} \pi_M(k, x_i, a_i)$. From (4), $\pi_M(k, x_i, I)$ is a decreasing function of μ_i and an increasing function of μ_j. We therefore get the following proposition.

Proposition 1. *We have* $\min_{M \in \mathcal{M}} \pi_M(k, x_i, I) = \pi_{M_{Wi}}(k, x_i, I)$, *where the worst case* M_{Wi} *for player i is given by* $\mu_i = \eta$ *and* $\mu_j = -\eta$.

Player i chooses I if and only if we have $\pi_{M_{Wi}}(k, x_i, I) > 0$, where, from (4) and Proposition 1, we have

$$\pi_{M_{Wi}}(k, x_i, I) = x_i - \eta - \Phi\left(\frac{k - x_i}{\sqrt{2}\sigma} + \sqrt{2}\frac{\eta}{\sigma}\right). \quad (5)$$

As, from (5), $\pi_{M_{Wi}}(k, x_i, I)$ is a strictly increasing function of x_i and a strictly decreasing function of k, there is a unique value $b(k)$, which strictly increases with k, such that $\pi_{M_{Wi}}(k, b(k), I) = 0$, and we have $\pi_{M_{Wi}}(k, x_i, I) > 0$ if $x_i > b(k)$, and $\pi_{M_{Wi}}(k, x_i, I) < 0$ if $x_i < b(k)$. This implies that the switching strategy with switching point $b(k)$ is a best response of player i to the switching strategy with switching point k of player j, and that the switching strategy with switching point k is an equilibrium of the game if and only if k is a solution of the equation $b(k) = k$, or equivalently $\pi_{M_{Wi}}(k, k, I) = 0$. Using (5) and solving this equation, we immediately get the following proposition.

Proposition 2. *There is a unique equilibrium in switching strategy.*[7] *The equilibrium switching point* k^* *is given by*

$$k^* = \eta + \Phi\left(\sqrt{2}\frac{\eta}{\sigma}\right). \quad (6)$$

In the special case of no ambiguity ($\eta = 0$), (6) gives $k^* = \Phi(0) = 1/2$, which is what is found in Morris and Shin (2003).

4. Effect of uncertainty

Uncertainty is characterized by the two parameters σ and η, which represent the "risk" and "ambiguity" components, respectively. From (6), we have $\partial k^*/\partial \eta > 0$, and therefore greater ambiguity increases k^*. There are two channels, going in the same direction. First, an increase in ambiguity raises the highest possible value η of the bias μ_i. This leads each player to infer a lower conditional expected value of θ in the worst case. This reduces the incentive to play I, which raises k^*.

The second channel is given by the term $\Phi\left(\sqrt{2}(\eta/\sigma)\right)$ in (6). The value $1 - \Phi\left(\sqrt{2}(\eta/\sigma)\right)$ is the conditional probability, in the

3 Cheli and Della Posta (2007) have argued that it may be legitimate to introduce some bias in the signal received.

4 Alternatively, rather than using such an improper prior for θ, we could have assumed that θ and ε_i are uniformly distributed.

5 When we apply the model to real world issues, as in the case of financial crises, such an assumption seems to be realistic. Each player may have some personal expert or staff of experts who give advices. Although there are public sources of information, each player may give to them a selective attention, which differs from one player to the other. Thus, all players do not necessarily read the same published documents, or get informed from the same media. They may weight differently these sources of information and the various opinions expressed in them. Each player may also have access to a different private, partial, and therefore possibly biased, piece of information.

6 This is a standard criterion (Gilboa and Schmeidler, 1989).

7 Along the same lines as in Morris and Shin (2003), it could also be shown that this equilibrium is the unique equilibrium which survives the iterated deletion of interim-dominated strategies.