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An alternative identification of nonlinear dynamic panel data models with unobserved covariates

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HIGHLIGHTS

- I consider the nonparametric identification of nonlinear dynamic panel data models.
- I achieve identification without specifying the distribution of the initial condition.
- I relax the assumption of covariate evolution in Shiu and Hu (2013).
- These models allow a direct feedback from the dependent variable on the future covariate.

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ABSTRACT

I provide the nonparametric identification of nonlinear dynamic panel data models. I relax the assumption of covariate evolution in Shiu and Hu (2013) by the results of Hu and Shum (2012). The assumptions include first-order Markov assumptions and a restriction on the evolution of the covariate.

1. Introduction

The study of dynamic panel data models for a fixed number of periods has been a leading topic in the econometrics literature because models of this form are appropriate for several applications (see Baltagi, 2008; Wooldridge, 2010; Hsiao, 2003). The estimation of linear dynamic panel data models can be performed within the GMM framework by applying a transformation to eliminate the unobserved effect and then constructing moment conditions. This approach typically requires the estimators suggested by Anderson and Hsiao (1982), Arellano and Bond (1991), Arellano and Bover (1995) and Ahn and Schmidt (1995).

In this paper I concentrate on the identification of nonlinear dynamic panel data models. Nonlinear models are more challenging than linear models because researchers do not know how to remove the unobserved effect from nonlinear models. Discrete choice logit models have been studied by Chamberlain (1980, 1984), and Honoré and Kyriazidou (2000), all of whom have considered a conditional probability approach for logit models to eliminate the unobserved effect. Shiu and Hu (2013) proposed a new identification method for nonlinear dynamic panel data models without making assumptions regarding the distribution of the initial condition. Their approach uses the spectral decomposition of linear operators and provides nonparametric identification of nonlinear dynamic panel data models using two periods of the dependent variable Y_{it} and three periods of the covariate X_{it}. The method relies on the assumption that the dynamic process of the future covariate X_{it+1} is independent of the variables Y_{it}, Y_{it-1}, and X_{it-1} conditional on X_{it} and U_{it}. However, ruling out a direct feedback from the dependent variable Y_{it} on the future covariate X_{it+1} conditionally may be strong in some empirical modeling. Allowing the dependent variable to have a direct impact on the next period covariates is important for a large range of empirical research. Let V be a city unobserved heterogeneity and ε_t be a serially correlated macro

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shock. Consider a simple dynamic panel data model to determine the effect of police officers per capita ($polpc_t$) on a city's murder rate ($murdrate_t$): $murdrate_t = g\left(polpc_t, murdrate_{t-1}, V + \varepsilon_t\right)$. If the cities adjust the size of their police force based on past values of the murder rate, then the dependent variables $murdrate_t$ should have a direct effect on the future covariate $polpc_{t+1}$. Another example is a model capturing how per capita condom sales in city affects the HIV infection rate: $hiv_t = g\left(conpc_t, hiv_{t-1}, V + \varepsilon_t\right)$, where hiv_t is the HIV infection rate during year t and $conpc_t$ be per capita condom sales for year t. It is highly likely that the future condom usage $conpc_{t+1}$ is influenced by the spread of HIV hiv_t. This paper presents a solution to sidestep the restriction.

I show that the identification strategies in Hu and Shum (2012) for dynamic models with unobserved state variables can be extended to establish nonparametric identification in nonlinear dynamic panel data models without specifying the distribution of initial conditions and allowing for a feedback from Y_{it} on X_{it+1}. The methods also relies on the unique eigenvalue–eigenfunction decomposition of an integral operator associated with joint densities of observable variables and the unobserved covariate in different periods. The conditional distribution of interest is $f_{Y_t|X_t,Y_{t-1},U_t}$ and the conditional density can be nonparametrically identified from $\left\{X_{t+1}, (Y_t, X_t), (Y_{t-1}, X_{t-1}), X_{t-2}\right\}$. Therefore, compared with the results in Shiu and Hu (2013), the method developed in this paper requires more periods of data on the covariate X_t. My main identifying assumptions include first-order Markov assumptions and some measure of location of the evolution of the covariate (e.g. its mean, mode or median) which is equal to zero.

I show the identification assumptions and results of nonlinear dynamic panel data models and their corresponding average partial effects in Section 2. Section 3 concludes. A detailed derivation of the identification will be presented in the Appendix.

2. Identification and assumptions

In this paper I consider the following nonlinear dynamic panel data model specification:

$$Y_t = g\left(X_t, Y_{t-1}, V + \varepsilon_t\right), \quad \forall t = 1, \ldots, T, \tag{1}$$

where g is an unknown function, V is a fixed effect and ε_t is an idiosyncratic shock that may be serially correlated.[1] Assume $V + \varepsilon_t = U_t + \xi_t$, where U_t is an unobserved covariate correlated with other observed explanatory variables (X_t, Y_{t-1}), and ξ_t stands for a random shock independent of all other explanatory variables (X_t, Y_{t-1}, U_t). Then, the proposed dynamic panel data model in Eq. (1) can be derived as a conditional distribution of the dependent variable of interest, Y_t, conditional on lagged values of that variable, a possibly time-varying explanatory variable X_t, and the unobserved covariate U_t, i.e., $f_{Y_t|X_t,Y_{t-1},U_t}$. As discussed in Shiu and Hu (2013), dynamic discrete choice models and dynamic censored models can be used to illustrate this identification connection. Here, I introduce dynamic Poisson models which can be written as $f_{Y_t|X_t,Y_{t-1},U_t}$ directly.

Example 2.1 (*Dynamic Poisson Model with Unobserved Covariate*). Consider

$$f_{Y_t|X_t,Y_{t-1},U_t} = \frac{\exp(-m(X_t, Y_{t-1}, U_t))m(X_t, Y_{t-1}, U_t)^y}{y!} \quad \text{with } y = 0, 1, \ldots,$$

where $E(Y_t|Y_{t-1}, \ldots, Y_1, X_t, \ldots, X_1, U_t) = m(X_t, Y_{t-1}, U_t)$ is an unknown mean function.

The general specification here allows us to cover a number of nonlinear dynamic panel data models under one framework. The specifications in Eq. (1) allow for three sources of persistence after controlling for the observed explanatory variable X_t, serial correlation in the transitory error term ε_t, unobserved individual heterogeneity V, and the true state dependence through the term Y_{t-1}. Distinguishing the sources of persistence is important because a policy that temporarily increases the probability of the event will have different implications about future probabilities of experiencing an event.

The first step of the identification approach is to nonparametrically identify the evolution of the future covariate:

$$f_{X_{t+1}|Y_t,X_t,U_t}.$$

The identification of the above density leads to that of the conditional density $f_{Y_t|X_t,Y_{t-1},U_t}$ which is connected to the proposed dynamic panel data models in Eq. (1). Below, I introduce assumptions and the main nonparametric identification results. The detailed derivation of the identification is presented in the Appendix. My identification scheme follows the intuition of the identification techniques in Hu and Shum (2012) and Shiu and Hu (2013).[2] Assume

Assumption 2.1 (*Exogenous Shocks*). The random shock ξ_t is independent of ξ_τ for any $\tau \neq t$ and $\{X_\tau, U_\tau\}$ for any $\tau \leq t$.

This assumption implies that I can derive the conditional density $f_{Y_t|X_t,Y_{t-1},U_t}$ from a variety of dynamic panel data models in Eq. (1),[3] but the assumption is not required for the dynamic Poisson model in Example 2.1.

Assumption 2.2. Denote $W_t = (Y_t, X_t)$. (i) (Markov covariate evolution) The evolution of the observed covariate satisfies the equation $f_{X_{t+1}|W_t,W_{t-1},X_{t-2},U_t} = f_{X_{t+1}|W_t,U_t}$; (ii) First-order Markov transition:

$$f_{W_{t+1}|W_t,U_t,\Omega_{<t}} = f_{W_{t+1}|W_t,U_t}. \tag{2}$$

where $\Omega_{<t} = (Y_{t-1}, X_{t-1}, \ldots, Y_1, X_1)$.

Assumption 2.2(i) and (ii) assume first-order Markov transitions over X_t and W_t, which are typical conditions for dynamic panel data models. This is the major difference and improvement from the corresponding assumption in Shiu and Hu (2013). The covariate evolution assumption in Shiu and Hu (2013) is $f_{X_{t+1}|Y_t,X_t,Y_{t-1},X_{t-1},U_t} = f_{X_{t+1}|X_t,U_t}$ which limits effects from the past dependent variables.[4] Let $\mathcal{W}_t$, $\mathcal{X}_t$, and $\mathcal{U}_t$ be the supports of the random variables W_t, X_t, and U_t, respectively. Set $1 \leq p < \infty$, and let $\mathcal{L}^p(\mathcal{X})$ stand for the space of functions $h(\cdot)$ with $\int_{\mathcal{X}} |h(x)|^p dx < \infty$. Given (w_t, w_{t-1}), define operators as follows:

$$L_{X_{t+1},w_t,w_{t-1},X_{t-2}} : \mathcal{L}^p(\mathcal{X}_{t-2}) \to \mathcal{L}^p(\mathcal{X}_{t+1})$$

$$(L_{X_{t+1},w_t,w_{t-1},X_{t-2}}h)(\tilde{x}) = \int f_{X_{t+1},W_t,W_{t-1},X_{t-2}}(\tilde{x}, w_t, w_{t-1}, x)h(x)dx,$$

$$D_{w_t|w_{t-1},U_t} : \mathcal{L}^p(\mathcal{U}_t) \to \mathcal{L}^p(\mathcal{U}_t) \text{ such that } (D_{w_t|w_{t-1},U_t}h)(u) = f_{W_t|W_{t-1},U_t}(w_t|w_{t-1}, u)h(u).$$

[1] I can easily extend this to a multivariate X_t by considering a single-index with $X_t'\beta$.

[2] As it was already discussed in Hu and Shum (2012) and Shiu and Hu (2013) the assumptions are regarded as high-level as compared to other studies. However, the proposed models are more flexible and allow for being nonparametric.

[3] See Shiu and Hu (2013) for the use of this assumption on dynamic discrete choice models and dynamic censored models.

[4] Their results do not completely rule out the effect of past dependent variables on the future X_{t+1} because the unobserved covariate U_t may contain the information of past dependent variables.