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Intertemporal price discrimination in infinite horizon

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HIGHLIGHTS

- In infinite horizon, a credible durable-good monopolist chooses to discriminate.
- The intertemporal price schedule is ruled by a differential equation.
- The gain of discrimination depends on agents' relative patience.
- The model encompasses Stokey (1979) and Landsberger and Meilijson (1985) as polar cases.

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ABSTRACT

In infinite horizon, a credible durable-good monopolist may resort to intertemporal price discrimination. We provide an analytical characterization of his optimal price policy when consumers and the monopolist have different values for the trade because of distinct discount factors.

1. Introduction

Consider a monopolist who produces a durable good and sells it over time to a set of consumers. The well-known Coase (1972) conjecture states that consumers anticipate that the monopolist has an incentive to lower the price until the competitive level and will postpone buying until the price falls to this level. At the equilibrium, prices fall to that level in the twinkling of an eye.

Rational expectations and the inability of the monopoly to commit are the key ingredients that drive the result. When the firm is credible and can commit from the beginning to stick to some price scheme, Stokey (1979) showed that the static monopoly price is charged in each period, which guarantees positive profits; all sales occur at the beginning of the game.

This last result crucially relies on the assumption that the firm and consumers have the same discount factor; it no longer holds when agents have different intertemporal preferences. For instance, when the firm is more patient, she should be able to price-discriminate among consumers. This point was made by Landsberger and Meilijson (1985) who proved that in this case, intertemporal price discrimination (IPD) becomes profitable in a finite time horizon. First, the difference between discount factors accounts for non-transferability and generates intertemporal discrimination as soon as the firm is more patient than consumers. Second, this strategy turns out to generate higher revenues than uniform pricing (UP). Indeed, a time-decreasing price schedule makes high-valuation consumers buy relatively early provided that it is not profitable for them to wait for a lower price. Imposing such an incentive constraint is however less costly for a monopoly that is more patient than consumers.

We propose here to write down a model in infinite horizon and to determine analytically the optimal price policy. We allow for different intertemporal preferences: the firm and consumers

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may have distinct discount factors, which encompasses polar cases treated by the literature (where the monopolist is either infinitely patient or has the same impatience as consumers like in Stokey, 1979), and also improves the characterization of other cases, like the one treated by Landsberger and Meilijson (1985). We resort to variational methods to overcome the issue of non-transferability with respect to money between the firm and consumers: since agents have distinct intertemporal preferences, they do not value transfers identically, which complicates the resolution of the model. We provide an analytical characterization of the optimal price path thanks to an ordinary differential equation. Our framework contains previous models including the settings of Stokey (1979), Landsberger and Meilijson (1985) and Salant (1989). Finally, this note considers all possible configurations in terms of agents' relative patience.

The next section presents the model in the case when the monopoly is strictly more patient than consumers, though not infinitely patient. Section 3 treats the limits of the model, the cases where the monopoly is infinitely patient, as impatient as consumers, or more impatient than consumers. Section 4 derives the gain from discrimination and provides a welfare analysis. Section 5 concludes.

2. Model

Time is continuously measured on $[0; +\infty]$. A monopolist produces a durable good that does not depreciate over time. The marginal cost to produce the good is constant and normalized to zero. The firm discounts time t at rate ρ by $e^{-\rho t}$. Following Stokey (1979) and Landsberger and Meilijson (1985), we rule out time consistency issues and assume that the firm commits to charge a price policy announced at the outset. We restrict our attention to stationary strategies, *i.e.*, strategies that do not depend on past history.[1]

Consumers may purchase at most once. Their mass is constant and normalized to one. There is asymmetric information because the monopolist does not know consumers' private valuation θ for the good. The distribution of θ is common knowledge. Its density function $f(.)$ is continuously differentiable with respect to the Lebesgue measure and strictly positive almost everywhere over its support $\Theta = [0, \overline{\theta}] \subset \mathbb{R}^+$. Since the lowest reservation value is 0, at any positive price, including the best single price, *i.e.*, the static monopoly price p^m, not all consumers buy and thus the market is not covered. As is standard in this literature, we assume that the hazard rate is strictly increasing: $\frac{\partial}{\partial\theta}\left[\frac{f(\theta)}{1-F(\theta)}\right] > 0$, which guarantees that the static monopoly profit $p[1 - F(p)]$ is maximal at $\Pi^m = p^m[1 - F(p^m)]$ with $p^m = \frac{1-F(p^m)}{f(p^m)} \in (0, \overline{\theta})$.

If the firm could observe valuations, then she would extract the whole consumer surplus by charging $p(\theta) = \theta$, achieving first-best perfect-discrimination (PD) profits $\Pi^{\mathrm{PD}} = \Pi^{\mathrm{FB}} = \mathbb{E}\theta = \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta) d\theta$. Trade would occur immediately since time is costly to the monopolist.

When the firm cannot observe valuations, the problem consists in designing a direct truthful mechanism that implements the trade of one unit of the good. For this purpose, the monopoly may offer a menu of contracts made up of a price and a time of purchase $\{p(\theta), \tau(\theta)\}$. Even though the consumer decides of his time of purchase $\tau(\theta)$ given some price schedule $p(t)$, his optimization is already taken into account by the monopoly from incentive-compatibility. The firm can thus impose some schedule of purchase $\tau(\theta)$. Consumer θ reporting $\tilde{\theta}$ gets the utility:

$$U(\theta, \tilde{\theta}) = e^{-r\tau(\tilde{\theta})}[\theta - p(\tilde{\theta})].$$

This surplus $\theta - p(\tilde{\theta})$ is discounted by $e^{-r\tau(\tilde{\theta})}$, where r is his discount rate. For now, we assume that the monopolist is more patient than consumers: $\rho < r$. In Section 3, we discuss the cases $\rho = 0$, $\rho = r$ and $\rho > r$.

Consumers with valuation below a threshold θ_0 are excluded from the market, where θ_0 corresponds to the marginal consumer who is indifferent between purchasing or not:

$$p(\theta_0) = \theta_0. \tag{1}$$

A physical constraint imposes that the time of purchase be nonnegative:

$$\tau(\theta) \geq 0. \tag{2}$$

In a finite-horizon setting, this constraint would read: $\tau(\theta) \in [0, T]$.

Individual rationality imposes that the price is no higher than θ, and that the firm offers nonnegative prices:

$$p(\theta) \in [0, \theta] \quad (IR_\theta). \tag{3}$$

The mechanism is incentive-compatible if and only if:

$$\theta \in \arg\max_{\tilde{\theta}} U(\theta, \tilde{\theta}) \quad (IC_\theta).$$

The first-order condition writes: $\left.\frac{\partial U}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = 0$, *i.e.*:

$$\dot{p}(\theta) - r\,\dot{\tau}(\theta)\,p(\theta) = -r\,\dot{\tau}(\theta)\,\theta. \tag{4}$$

Given the time schedule $\tau(\theta)$, this incentive constraint looks like a differential equation in $p(.)$ with the initial condition (1). At the optimum, consumer θ enjoys the indirect utility

$$U(\theta, \theta) = V(\theta) \equiv \max_{\tilde{\theta}} e^{-r\tau(\tilde{\theta})}[\theta - p(\tilde{\theta})]. \tag{5}$$

The function $V(.)$ is the upper bound of a family of increasing affine functions of θ and hence is convex in θ. From the envelope theorem, $\dot{V}(\theta) = e^{-r\tau(\theta)}$ and is nondecreasing in θ. As a result, $\tau(\theta)$ is non-increasing in θ: high-valuations always buy before low-valuations. From Eq. (4), $\dot{p}(\theta) = -r\,\dot{\tau}(\theta)[\theta - p(\theta)] > 0$, the optimal price scheme is nondecreasing in valuations. Solving this equation yields an explicit relationship between $p(\theta)$ and $\tau(\theta)$:

$$p(\theta) = \theta - \int_{\theta_0}^{\theta} e^{r[\tau(\theta)-\tau(u)]} du. \tag{6}$$

The firm's program consists in maximizing profits discounted at rate ρ under individual rationality and incentive-compatibility constraints:

$$\max_{(p(\theta),\tau(\theta),\theta_0)} \int_{\theta_0}^{\overline{\theta}} e^{-\rho\tau(\theta)} p(\theta) f(\theta) d\theta \quad \text{s.t. (2), (3), (6),}$$

which, using (6), reduces to:

$$\max_{(\tau(\theta),\theta_0)} \int_{\theta_0}^{\overline{\theta}} e^{-\rho\tau(\theta)} \left[\theta - \int_{\theta_0}^{\theta} e^{r[\tau(\theta)-\tau(u)]} du\right] f(\theta) d\theta \quad \text{s.t. (2).}$$

The firm fixes the level of the marginal consumer type θ_0 optimally. She may decide not to sell to all consumers by choosing $\theta_0 > 0$. *De facto*, the first-order condition with respect to θ_0 imposes:

$$\theta_0 = \frac{\int_{\theta_0}^{\overline{\theta}} e^{(r-\rho)\tau(\theta)} f(\theta) d\theta}{e^{(r-\rho)\tau(\theta_0)} f(\theta_0)} = \frac{\int_{\theta_0}^{\overline{\theta}} w(\theta) f(\theta) d\theta}{f(\theta_0)} > 0, \tag{7}$$

[1] From Ausubel and Deneckere (1989), we know that non-stationary or history-dependent strategies enable the monopolist to do better than a Coasian outcome.