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On the efficient provision of public goods by means of biased lotteries: The two player case

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HIGHLIGHTS

- Biased lotteries can be used to achieve the efficient outcome in public good provision games.
- We characterize the minimal prize sum for the lottery necessary to implement the first-best.
- The minimal prize sum behaves non-monotonically as a function of consumers' heterogeneity.

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ABSTRACT

In this paper we analyze how biased lotteries can be used to overcome the free-riding problem in voluntary public good provision. We characterize the optimal combinations of bias and lottery prize and the conditions that guarantee efficient public good provision in equilibrium.

1. Introduction

Voluntary contributions to public good provision frequently result in inefficiently low levels of the public good due to free-riding of contributing agents. From a theoretical perspective free-riding occurs because contributing agents do not internalize the positive externalities from public good provision. Motivated by the frequently applied approach of charities in the real world, Morgan (2000) analyzed theoretically whether lotteries can be combined with voluntary contribution games to increase public good provision. His analysis revealed that specific types of self-financing lotteries induce negative externalities on participating agents which partially offset the positive externalities from public good provision and result in higher net amounts of the public good. Furthermore, he showed that public good provision is increasing in the prize sum. These results are also robust with respect to various modifications and extensions of the underlying model, compare Duncan (2002), Pecorino and Temimi (2007), and Lange (2006) for some examples. However, a caveat of most of these studies, as well as of Morgan (2000), is that no prize sum of finite value can induce the efficient allocation: while there are Pareto-improvements in comparison to the original voluntary contribution model without lotteries, there is still inefficient underprovision of the public good in equilibrium.[1]

We explicitly address this issue in our paper and show for the two player case that an appropriately biased lottery with a *finite* prize sum can induce the efficient level of public good provision if players are not identical.[2] More precisely, we characterize feasible

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[1] Kolmar and Wagener (2012), and Giebe and Schweinzer (2014) address this specific issue but both rely on coercive taxation to finance lottery prizes. In contrast, we retain the original assumption of Morgan (2000), where lottery prizes are completely financed out of lottery proceeds.

[2] The underlying intuition stems from Franke et al. (2013), where we analyzed the revenue-enhancing potential of optimally biased contest games. Whether biasing the lottery in a public good framework could induce efficient public good provision is not obvious ex-ante.

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combinations of prize sum and bias that induce the efficient level of public good provision in any interior equilibrium of the respective voluntary contribution game. Moreover, we derive the minimal prize sum that is necessary to induce the efficient level of public good provision and analyze its relation to the underlying players' heterogeneity. Finally, we provide conditions that guarantee the existence of interior and efficient equilibria such that both players decide voluntarily to contribute positive amounts to the public good.

The rest of the paper is organized as follows. In Section 2 we introduce the model and summarize previous results. In Section 3 we analyze the two player case, derive the optimal combinations of bias and lottery prize, and analyze implications. We conclude in Section 4 by discussing robustness and possible extensions. All proofs are relegated to an Appendix.

2. The model

There are n risk-neutral consumers with quasi-linear utility functions $u_i(w_i, G) = w_i + h_i(G)$, where w_i is the wealth of consumer i and G denotes the public good. It is standard to assume that $h_i' > 0$, $h_i'' < 0$, and $h_i'(0) > 1$ for all i, which implies that the public good is socially desirable. We also assume that wealth can be transformed one-to-one into the public good.

The efficient amount $\hat{G}$ of the public good satisfies the well-known optimality condition due to Samuelson (1954): $\sum_{i=1}^{n} h_i'(\hat{G}) = 1$.[3] However, private provision of the public good by voluntary contributions results in inefficient underprovision due to free-riding behavior of the consumers. Bergstrom et al. (1986) prove that the amount G^{BBV} of the public good provided in the equilibrium of the voluntary contribution game is always below the efficient level $\hat{G}$: $G^{BBV} < \hat{G}$.

Morgan (2000) modified the voluntary contribution game by introducing fixed-prize raffles, where a pre-announced fixed prize of value R is offered by the public good provider. Individual contributions to the public good, denoted by x_i, are awarded with lottery tickets on a one-to-one basis. Hence, consumer i's probability to win the prize is $\frac{x_i}{\sum_{j=1}^{n} x_j}$ (as long as $\sum_{j=1}^{n} x_j > 0$), which is a special case of a so-called contest success function introduced by Tullock (1980). The prize sum R itself is financed out of total contributions $\sum_{i=1}^{n} x_i$ such that only the remaining amount $\sum_{i=1}^{n} x_i - R$ can be used for public good provision. Consumer i consequently maximizes expected utility:

$$u_i(x_i, x_{-i}) = w_i - x_i + \frac{x_i}{\sum_{j=1}^{n} x_j} R + h_i\left(\sum_{j=1}^{n} x_j - R\right).$$

In the equilibrium of this modified voluntary contribution game involving lotteries the net amount G^M of public good provision (total contributions minus prize sum) is actually higher than in the original voluntary contribution game. However, underprovision of the public good still prevails because no finite prize sum R leads to the efficient amount $\hat{G}$ of the public good. The following proposition summarizes these results:

Proposition 2.1. *The voluntary contribution game with lotteries leads to a Pareto-improvement with respect to the original voluntary contribution game; however, for any finite prize sum underprovision of the public good prevails:* $G^{BBV} < G^M < \hat{G}$.

3. Biased lotteries: the two player case

We consider a voluntary contribution game with two consumers and biased lotteries, where the individual contribution x_i is weighted by parameter $\alpha_i > 0$ for $i = 1, 2$. The resulting winning probability $p_i = \frac{\alpha_i x_i}{\alpha_1 x_1 + \alpha_2 x_2}$ is therefore biased. As this contest success function is homogeneous of degree zero, we can normalize it such that $\alpha_1 = 1$ and $\alpha_2 = \alpha > 0$. This leads to the following expected utility functions:

$$u_1(x_1, x_2) = w_1 - x_1 + \frac{x_1}{x_1 + \alpha x_2} R + h_1\left(x_1 + x_2 - R\right) \tag{1}$$

$$u_2(x_2, x_1) = w_2 - x_2 + \frac{\alpha x_2}{x_1 + \alpha x_2} R + h_2\left(x_1 + x_2 - R\right). \tag{2}$$

For this modification of the Morgan (2000) setup existence and uniqueness of the equilibrium are preserved. We assume in the following that the equilibrium in the two player game is interior[4] and can therefore be characterized by the respective first order conditions (second order conditions for a maximum always hold):

$$-1 + \frac{\alpha x_2^*}{(x_1^* + \alpha x_2^*)^2} R + h_1'\left(x_1^* + x_2^* - R\right) = 0, \tag{3}$$

$$-1 + \frac{\alpha x_1^*}{(x_1^* + \alpha x_2^*)^2} R + h_2'\left(x_1^* + x_2^* - R\right) = 0. \tag{4}$$

Our objective is to show that there exist bias–prize combinations (α, R) such that the resulting equilibrium allocation (x_1^*, x_2^*) induces the efficient level $\hat{G}$ of public good provision; i.e., Eqs. (3) and (4) must hold simultaneously with the Samuelson condition and total contributions must finance the prize sum of the lottery:

$$h_1'(\hat{G}) + h_2'(\hat{G}) = 1 \Longleftrightarrow x_1^* + x_2^* = \hat{G} + R. \tag{5}$$

The next result shows that consumer heterogeneity is a necessary condition for the existence of bias–prize combinations that induce efficient equilibria.

Proposition 3.1. *Identical consumers will not provide the efficient amount $\hat{G}$ of the public good in any voluntary contribution game with biased lotteries independently of the respective bias–prize combination (α, R).*

We therefore concentrate on non-identical consumers in the following sense:

Definition 3.2. Two consumers are heterogeneous if $h_1'(\hat{G}) \neq h_2'(\hat{G})$. Heterogeneity is measured by parameter $\hat{h} = \frac{1-h_2'(\hat{G})}{1-h_1'(\hat{G})} > 0$.

Measure $\hat{h}$ is positive (implied by Eqs. (3) and (4)) and only depends on the preference parameters because they determine the efficient level $\hat{G}$ of public good provision.

The main result of the paper is presented in the following theorem which states that the lottery can be biased such that the efficient level $\hat{G}$ of public good provision is induced in equilibrium, as long as consumers are heterogeneous and the equilibrium is characterized by first order conditions.

Theorem 3.3. *If consumers are heterogeneous (i.e. $\hat{h} \neq 1$) there always exist feasible combinations (α, R) of lottery prize sum and bias such that Eqs. (3)–(5) are simultaneously satisfied.*

The exact relation between efficient prize sum and bias is presented in the proof and can be used to derive a lower bound on the prize sum that is, in combination with the corresponding bias, necessary to induce efficient public good provision.

3 As in Morgan (2000) we assume that wealth constraints are non-binding.

4 We discuss this assumption and its implications in detail ex-post.