Economics Letters 125 (2014) 447–450

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A nonparametric approach to solving a simple one-sector stochastic growth model

Philip Shaw *

Fordham University, Department of Economics, 441 East Fordham Rd., Bronx, NY 10458, United States

HIGHLIGHTS

- We introduce a nonparametric approach to solving nonlinear stochastic dynamic models.
- The distinct advantage of this approach is that there are no restrictions placed on the unknown conditional expectations function.
- This approach is shown to be stable and accurate when applied to a simple one-sector stochastic growth model.

ARTICLE INFO

Article history:
Received 28 April 2014
Received in revised form
17 September 2014
Accepted 15 October 2014
Available online 6 November 2014

JEL classification:
C63
C68

Keywords:
Nonparametric econometrics
Computational methods
Parameterized expectations algorithm

ABSTRACT

In this paper we present a nonparametric approach to solving a simple one-sector stochastic growth model. A distinct advantage of our approach is that it does not require placing restrictions on the generally unknown conditional expectations functions. Our method is shown to be accurate and computationally stable when compared to the standard Parameterized Expectations Approach (PEA) and the traditional linear approximation. We demonstrate this using a simple stochastic general equilibrium model with a known solution.

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1. Introduction

In this paper we introduce a nonparametric method for computing equilibria in nonlinear stochastic dynamic models. The distinct advantage to this approach is that there are no restrictions placed on the functional form of the underlying conditional expectations function to be estimated. We show that the method performs very well against both the traditional Parameterized Expectations Approach (PEA) introduced by Marcet (1988) and the traditional linear approximation.

2. A general framework

Following Maliar and Maliar (2003) we characterize the economy by a vector of n variables, z_t, and s exogenously determined shocks, u_t. Furthermore, let x_t be a subset of (z_{t-1}, u_t) and let the process $\{z_t, u_t\}$ be represented by the following system:

$$g(E_t[\phi(z_{t+1}, z_t)], z_t, z_{t-1}, u_t) = 0 \quad \text{for all } t \tag{1}$$

where $g\colon R^m \times R^n \times R^n \times R^s \to R^q$ and $\phi\colon R^{2n} \to R^m$. The conditional expectations function $E[\phi(z_{t+1}, z_t)|x_t] = \Phi(x_t)$ is generally unknown and thus the solution to the system is generally unknown. One approach to solving this problem is to use the method of Marcet (1988) which is known as the parameterized expectations approach (PEA). The basic idea behind the method is to approximate the conditional expectations functions by imposing parametric assumptions on the conditional expectations functions. This approach is viable because by definition the conditional expectations function will solely be a function of the conditioning set of variables. In theory, this means that one can select a function that can approximate the unknown functions with an arbitrary level of accuracy. The distinct advantage to using projection methods, such as the PEA, is that it is easy to implement in practice. Furthermore, they can be much faster than more accurate measures such as value function iteration or methods that rely upon numerical integration, especially when the dimension of the state space is

* Tel.: +1 618 806 4988.
E-mail address: pshaw5@fordham.edu.

http://dx.doi.org/10.1016/j.econlet.2014.10.011
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large. In addition to this, projection methods do not suffer from the curse of dimensionality in the same way as value function iteration. This same advantage will be true for the nonparametric approach we lay out in this paper. The disadvantage of the PEA is that it can exhibit implosive or explosive behavior. This is a problem specifically addressed by Maliar and Maliar (2003) where they show that by placing moving bounds on the endogenous state variables, one can overcome the implosive or explosive behavior of the PEA algorithm. The basic idea of PEA can be summarized as follows:

$$\beta^* = \underset{\beta \in R^v}{\operatorname{argmin}} \|\psi(\beta, x_t) - \Phi(x_t)\| \tag{2}$$

where the PEA seeks to find a vector of parameters that minimize the distance between the actual expectations function and a fixed approximate. One problem with this approach, as with any parametric approach, is that the approximating function can be misspecified resulting in an inaccurate solution. In practice, one could just increase the order of approximation however this approach can be costly computationally as the order of expansion increases to reduce the degree of model misspecification. Furthermore as shown by Judd et al. (2011) higher order approximations lead to an ill-posed inverse problem when using standard parametric methods to estimate the conditional expectations. They address the stability problem by offering a wide variety of parametric approximation methods, including regularization methods, which greatly increase the accuracy and stability of the parameterized expectations approach.

Our approach differs from the traditional PEA in that we avoid function selection and the ill-posed problem directly. Using a nonparametric approach, we focus on estimating the conditional expectations function directly using the joint and marginal distributions of the variables given by the following expression[1]:

$$E[\phi(z_{t+1}, z_t)|x_t] = \int \frac{\phi(z_{t+1}, z_t) f(z_{t+1}, z_t, x_t)}{f(x_t)} dz_{t+1} dz_t. \tag{3}$$

In theory, the conditional expectations operator is the best predictor in the mean squared error sense. Thus there exists no other function that predicts ϕ better than Eq. (3). Only under certain assumptions will a given parametric function coincide with the conditional expectations function. In practice, one can estimate this conditional expectations function nonparametrically using the generalized product kernel as presented in Li and Racine (2003):

$$\begin{aligned}\hat{E}[\phi(z_{t+1}, z_t)|x_t = x] &= \hat{m}(x) \\ &= \frac{\sum_{t=1}^{T} \hat{\phi}_t \prod_{i}^{r_1} \frac{1}{h_i} w\left(\frac{x_i^c - x_{it}^c}{h_i}\right) \prod_{i}^{r_2} \lambda_i^{I(x_{it}^d \neq x_i^d)} \prod_{i}^{r_3} \lambda_i^{|x_{it}^s - x_i^s|}}{\sum_{t=1}^{T} \prod_{i}^{r_1} \frac{1}{h_i} w\left(\frac{x_i^c - x_{it}^c}{h_i}\right) \prod_{i}^{r_2} \lambda_i^{I(x_{it}^d \neq x_i^d)} \prod_{i}^{r_3} \lambda_i^{|x_{it}^s - x_i^s|}}\end{aligned} \tag{4}$$

where x^c are continuous state variables, x^d are discrete state variables, and x^s are discrete variables with a natural ordering. Given the general form for our product kernel, the nonparametric approach can handle any type of state variable whether they be continuous or discrete. We select the bandwidth parameter using the rule-of-thumb: $h = c\left[\prod_{i=1}^{r_1} \sigma_i\right]^{1/r_1} T^{-\frac{1}{4+r_1}}$ where $c \in R^+$ and σ_i is the standard deviation of x_i. Other choices for bandwidth selection are cross-validation or modified Akaike information criterion (AIC) procedures.[2] We use the following update method for the choice of bandwidth $h^{j+1} = (1-\omega)h^j_{pilot} + \omega\hat{h}$ where $\hat{h}$ is the $j+1$ bandwidth calculated via ROT with $\omega \in (0, 1]$. This is similar to the homotopy approach followed by Marcet (1988).

3. Implementation

To implement the nonparametric expectations approach (NPEA) in practice we must start off with an initial guess for the conditional expectations function. Given this we can calculate the sequence of variables z_t. Then we update the estimate for the conditional expectations function and iterate until convergence.

- Step one. Given a sequence of $\{u_t\}_{t=1}^T$ and initial guess $\hat{m}^i$, calculate $\{z_t, \hat{\phi}_t\}_{t=1}^T$.
- Step two. Given the sequence of endogenous and exogenous state variables generated above update the conditional expectations functions to $\hat{m}^{i+1}$.
- Step three. Check for convergence of the conditional expectations function such that $D(\hat{m}^i, \hat{m}^{i+1}) < \epsilon$ for some distance function D.

To initialize the algorithm we use a parametric function with an uninformative prior so that our starting function is given by $\psi(\beta = 0; x_t)$. This assumption allows the researcher to remain agnostic about the underlying function to be estimated. As shown later this initialization does not affect the convergence of the nonparametric method.

4. An example

We follow the example as laid out by Maliar and Maliar (2003) and Duffy and McNelis (2001) where they consider a simple one-sector stochastic growth model. The basic setup is as follows:

$$\max_{\{c_t, k_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \text{s.t. } c_t + k_t = (1-d)k_{t-1} + \theta_t k_{t-1}^{\alpha} \tag{5}$$

where $\theta_t = \theta_{t-1}^{\rho} \exp(u_t)$ and $u_t \overset{d}{\sim} N(0, \sigma^2)$ From the first order conditions we obtain the classic Euler equation presented as:

$$c_t^{-\gamma} = \delta E\left[\frac{1 - d + \theta_{t+1}\alpha k_t^{\alpha-1}}{c_{t+1}^{\gamma}}\middle|\theta_t, k_{t-1}\right]. \tag{6}$$

Just by the nature of the conditional expectation function, Eq. (6) will only be a function of the conditioning set. In general, the solution to Eq. (6) cannot be found analytically. However when $\gamma = 1$ and $d = 1$ we can show that $c_t = (1 - \alpha\delta)\theta_t k_{t-1}^{\alpha}$. Using the PEA we might try to approximate the unknown conditional expectations with the following approximate:

$$\begin{aligned}\psi(\beta; \theta_t, k_{t-1}) = \exp(&\beta_0 + \beta_1 \log(\theta_t) + \beta_2 \log(k_{t-1}) \\ &+ \beta_3(\log(k_{t-1}))^2 + \beta_4(\log(\theta_t))^2 \\ &+ \beta_5 \log(k_{t-1}) \log(\theta_t)).\end{aligned} \tag{7}$$

Once the functional form is chosen and the parameter vector β^{i-1} is initialized, then a sequence of $k_t(\beta^{i-1}) = \psi(\beta^{i-1}; k_{t-1}(\beta^{i-1}), \theta_t)$ and $c_t(\beta^{i-1}) = (1 - d)k_{t-1} + \theta_t k_{t-1}^{\alpha} k_t(\beta^{i-1})$ is generated for an exogenous sequence of θ_t. Once this is done, the researcher

[1] A similar approach has been developed by Jirnyi and Lepetyuk (2011) but their focus is particularly on solving for the dynamics of heterogeneous agent models with aggregate uncertainty. Their approach also relies on an alternative to the kernel methods presented in this paper where they use a K-nearest neighborhood approach instead of the local constant approach presented in this paper.

[2] For a detailed treatment of these methods see Racine and Li (2007).

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