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Tightening bounds in triangular systems

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HIGHLIGHTS

- We study partial identification in a nonparametric triangular system.
- We consider a triangular system with discrete endogenous regressors.
- We propose a simple idea that allows to improve existing bounds in the literature.

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ABSTRACT

This note discusses partial identification in a nonparametric triangular system with discrete endogenous regressors and nonseparable errors. Recently, Jun et al. (2011, JPX) provide bounds on the structural function evaluated at particular values using exclusion, exogeneity and rank conditions. We propose a simple idea that often allows to improve the JPX bounds without invoking a new set of assumptions. Moreover, we show how our idea can be used to tighten existing bounds on the structural function in more general triangular systems.

1. Introduction

In this note, we consider the following nonparametric triangular model:

$$\begin{cases} Y = g(D, U) \\ D = h(Z, V) \end{cases} \tag{1}$$

where $Y \in \mathcal{Y} \subset \mathbb{R}, D \in \mathcal{D} \subset \mathbb{R}^d, Z \in \mathcal{Z} \subset \mathbb{R}^{d_z}$ are observables and g and h are unknown functions with g nondecreasing in U and left-continuous for all values of D and $h_j(Z, V_j)$ the jth component of the vector $h(Z, V) = [h_1(Z, V_1), \ldots, h_d(Z, V_d)]$ nondecreasing in V_j and left-continuous for all values of Z, for $j = 1, \ldots, d$. $U \in \mathcal{U} = (0, 1]$, $V \in \mathcal{V} \subseteq \mathcal{U}^d$ are errors. We refer to D as endogenous regressors and Z as instruments which need not be continuous. Our objective is identification of the object

$$\psi^* = \psi(d^*, \tau^*, v^*) = g(d^*, Q_{U|V}(\tau^*|v^*)) \tag{2}$$

for given values of $(\tau^*, d^*, v^*) \in \mathcal{U} \times \mathcal{D} \times \mathcal{V}$, where $Q_{U|V}(\tau|v) = \inf\{u : \mathbb{P}(U \leq u|V = v) \geq \tau\}$. A model similar to (1) has been studied in Chesher (2003, 2005) and in Jun et al. (2011, JPX). Chesher (2003) used an assumption of strict monotonicity to identify the partial derivatives of g with respect to D. However, when D is discrete the strict monotonicity assumption does not hold and then fails to point identify the quantity of interest ψ^*. Therefore, Chesher (2005) proposed to bound ψ^* under a dependence condition on U and V, as well as "local exclusion", and "local exogeneity" conditions on the instrument Z. JPX proposed the use of "global" rather than "local" conditions in the sense that they imposed a global exclusion restriction (Z does not enter g) and assumed that Z is independent of (U, V). Although their global conditions are stronger than the Chesher (2005) local ones, they have some interesting advantages. First, the global conditions allow them to replace a rank condition in Chesher (2005) with

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an alternative weaker rank condition that in some cases permits the construction of tighter bounds on ψ^* than those obtained in Chesher (2005). Second, this weaker rank condition allows them to construct meaningful bounds on ψ^* when D is binary something that Chesher (2005) cannot do. Therefore, JPX proposed a general method to derive tighter bounds on ψ^* under a set of global conditions.

In this note, we propose a simple idea that allows us to tighten the JPX bounds without invoking a new set of assumptions. Indeed, we show that the weak monotonicity and left-continuity assumptions imposed on both g and h_j plus the global conditions allow identification of the sign of $[\psi(d,\tau^*,v^*)-\psi(d',\tau^*,v^*)]$ for $(d,d')\in(\mathcal{D}\times\mathcal{D})$ in some cases. We show how this new information can help tighten the bounds proposed by JPX. The JPX method uses variation in the instrument to provide meaningful bounds on ψ^*. In addition to their strategy, we propose to use variation in D (across treatment) to tighten their bounds and then propose sharper bounds on ψ^*. For instance, we show that whenever Y, D, and Z are binary, the JPX bounding approach may fail to provide meaningful lower or upper bounds for either $\psi(1,\tau^*,v^*)$ or $\psi(0,\tau^*,v^*)$ while our strategy does.

For the sake of simplicity, we initially consider a simple case where Y and D are both binary and generalize our argument later. We only show the improvement that we can obtain on the JPX bounds when D is binary, but this improvement would become more important when D takes multiple values or/and in the presence of other exogenous covariates that enter both g and h.

The rest of the note is organized as follows. In Section 2, we consider a simple binary triangular case of model (1). This simple case helps us illustrate ideas and demonstrate the improvement obtained on the JPX bounds using our approach. Section 3 discusses the generalization of our argument for the nonbinary triangular system. The last section concludes.

2. Simple case: binary triangular system

We adopt, without loss of generality (w.l.o.g), the framework of the potential outcomes model $Y=Y_1D+Y_0(1-D)$, where $Y_d=g(d,U), d\in\{0,1\}$ are binary unobserved potential outcomes. Since $g(d,U)$ is weakly increasing in its second argument and U is uniform on $[0,1]$, we have $g(d,u)=\inf\{y:\mathbb{P}(Y_d\le y)\ge u\}$, which is equal to $1\{\mathbb{P}(Y_d=0)<u\}$ since Y_d is binary. Left-continuity holds, because $\mathbb{P}(Y_d\le y)$ is a cadlag function of y. Therefore, the binary triangular system can be written w.l.o.g as follows:

$$\begin{cases} Y=1\{\vartheta(D)<U\}\\ D=1\{p(Z)<V\}\end{cases} \tag{3}$$

where $U,\ V\sim\mathcal{U}[0,1]$. Then, we have $\vartheta(d)=\mathbb{P}(Y_d=0)$ and $p(Z)=\mathbb{P}(D=0|Z)$.

The formal assumptions we use in this section may be expressed as follows:

Assumption 1. (U,V) are independent of Z.

Assumption 2. U is positive regression dependent on V, i.e. $Q_{U|V}(\tau|v)$ is nondecreasing in v for all values of τ.

Assumption 3. $\mathcal{Z}(d^*,v^*)=\{z\in\mathcal{Z}:1\{p(z)<v^*\}=d^*\}$ is nonempty for $d^*\in\{0,1\}$.

This latter assumption ensures observation of individuals in both treatment groups i.e $D=0$ and $D=1$ when $V=v^*$. The assumptions made above are presumed to hold throughout the rest of the paper. Under Assumptions 1–3, Lemma 1 in JPX states that $\psi(d^*,\tau^*,v^*)=Q_{Y|D,V}(\tau^*|d^*,v^*)$. Then $\psi(d^*,\tau^*,v^*)=\inf\{y:\mathbb{P}(Y\le y|D=d^*,V=v^*)\ge\tau^*\}=1\{\mathbb{P}(Y_{d^*}=0|D=d^*,V=v^*)<\tau^*\}$. The last equality holds since Y_d is binary. Note that, under Assumption 3, there exists $z^*\in\mathcal{Z}$ such that $\mathbb{P}(Y_{d^*}=0|D=d^*,V=v^*)=\mathbb{P}(U\le\vartheta(d^*)|Z=z^*,V=v^*)$, which is equal to $\mathbb{P}(U\le\vartheta(d^*)|V=v^*)$ under Assumption 1. Then, in this special case our function of interest is

$$\psi^*=1\left\{\mathbb{P}(U>\vartheta(d^*)|V=v^*)>1-\tau^*\right\},\quad d^*\in\{0,1\}. \tag{4}$$

Therefore, the main issue is to provide the tightest bounds for $\mathbb{P}(U>\vartheta(d^*)|V=v^*)$. For the sake of clarity, we will explain briefly the bounding approach proposed by Chesher (2005) and JPX before presenting our refinement. In the rest of the paper, we shall use the following notations $p(z)=\mathbb{P}(D=0|Z=z)$, $\mathbb{P}(1|1,z)=\mathbb{P}(Y=1|D=1,Z=z)=\mathbb{P}(U>\vartheta(1)|V>p(z))$, and $\mathbb{P}(1|0,z)=\mathbb{P}(Y=1|D=0,Z=z)=\mathbb{P}(U>\vartheta(0)|V\le p(z))$ for $z\in\mathcal{Z}$. Now, we are going to present two special cases of different rank conditions to illustrate our idea.

First illustrative case: The support of Z contains four distinct values i.e. $\mathcal{Z}=\{z_1,z_2,z_3,z_4\}$ such that $0<p(z_1)<p(z_2)<v^*<p(z_3)<p(z_4)<1$.

2.1. Chesher (2005)'s bounding approach

Assumption 2 implies that $\mathbb{P}(U>\vartheta(1)|V=v)$ is nondecreasing in v, then we have:

$$\begin{aligned}&\mathbb{P}(U>\vartheta(1)|V=v^*)\\ &\quad\le\mathbb{P}(U>\vartheta(1)|V=v)\quad\text{for } v\in[p(z_i),1],\ i=3,4,\\ &\mathbb{P}(U>\vartheta(1)|V=v^*)(1-p(z_i))\\ &\quad\le\int_{[p(z_i),1]}\mathbb{P}(U>\vartheta(1)|V=v)dv\quad\text{for } i=3,4.\end{aligned}$$

The second inequality holds by taking the integral over both parts. The last inequality implies that:

$$\begin{aligned}&\mathbb{P}(U>\vartheta(1)|V=v^*)\\ &\quad\le\frac{1}{1-p(z_i)}\int_{[p(z_i),1]}\mathbb{P}(U>\vartheta(1)|V=v)dv\quad\text{for } i=3,4,\\ &\quad=\mathbb{P}(1|1,z_i)\quad\text{for } i=3,4.\end{aligned}$$

Therefore,

$$\mathbb{P}(U>\vartheta(1)|V=v^*)\le\min\ (\mathbb{P}(1|1,z_3),\mathbb{P}(1|1,z_4)). \tag{5}$$

Notice that, this cannot be done for z_1 and z_2 since $v^*\in[p(z_i),1]$ *for* $i=1,2$. However, we can similarly derive the following:

$$\begin{aligned}&\max\ (\mathbb{P}(1|0,z_1),\mathbb{P}(1|0,z_2))\\ &\quad\le\mathbb{P}(U>\vartheta(0)|V=v^*)\quad\text{for } i=1,2.\end{aligned} \tag{6}$$

Note that, using the idea behind Chesher (2005), we cannot provide meaningful lower and upper bounds respectively for $\mathbb{P}(U>\vartheta(1)|V=v^*)$ and $\mathbb{P}(U>\vartheta(0)|V=v^*)$. JPX introduced an idea that allows them to refine those bounds and provide meaningful bounds when the latter approach fails.

2.2. JPX's bounding approach

The Chesher (2005) idea exploits information from the different intervals $[p(z_i),1]$ and $[0,p(z_i)]$. JPX pointed out that it is also possible to exploit information from $[p(z_1),p(z_2)]$ and $[p(z_3),p(z_4)]$. Indeed, for $p(z_i)<p(z_j)$, we can show the following