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Multiplicity of monetary steady states

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HIGHLIGHTS

- We study a variant of the Lagos–Wright framework.
- We assume that the decentralized market opens twice in each period.
- We show that there may be multiple equilibria.
- The multiplicity comes from the nonconcavity of the value function.

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ABSTRACT

We investigate a monetary model in which the centralized market opens once, but the decentralized markets open twice in each period. We show that there may be multiple stationary equilibria.

1. Introduction

The microfoundation of monetary models has been an important topic in macroeconomics. In a seminal paper by Lagos and Wright (2005), henceforth LW, a tractable monetary model is constructed in which the role of money is explicitly described. In LW, each period is divided into two subperiods, day and night. Money facilitates trades in the day market in which buyers and sellers are anonymous. The LW framework is now heavily used in monetary economics.

In LW, the uniqueness of the steady state is shown only under specific restrictions on the utility function.[2] Recently, Wright (2010) relaxed the assumption and showed that the steady state is always unique. Wright (2010) also argued that monetary frictions alone cannot generate multiple equilibria.

In this note, we study a version of LW in which decentralized markets open *twice* in each period. Each date is divided into three subperiods—morning, afternoon, and night. The morning and afternoon markets are decentralized, while the night market is centralized. We show that for some utility functions, there are multiple stationary equilibria. This is because the objective function of the buyer in the morning market depends on the seller's money balances, and then the first-order conditions may not be monotone in the equilibrium money balances. The uniqueness result in Wright (2010) depends on the assumption that the agents always enter the centralized market after the transactions in the decentralized market.[3]

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2 LW shows the uniqueness when the utility function in the day market u satisfies $u'u''' \leq (u'')^2$.

3 Lagos and Wright (2003) studied a model which is similar to LW, but they showed that the steady states can be multiple. The difference between the two papers is that the former introduce real return on money, while the latter does not. Our model closely follows LW.

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Our set-up is similar to that of Ennis (2009), who investigated a variant of LW by assuming that buyers may bypass the centralized market; however, he does not study equilibrium multiplicity. Our model is also close to Berentsen et al. (2005) in which DMs open twice each period. They show the equilibrium uniqueness for some parameter values, but they do not investigate the equilibrium multiplicity.

2. Model

2.1. Set-up

Time is discrete and changes from $t = 0$ to $+\infty$. There is a continuum of infinitely lived agents with unit measure. Each date is divided into *three* subperiods—morning, afternoon, and night. The morning and afternoon markets are decentralized and the night market is centralized. In the following, consecutive period variables are indexed by $+1$.

At the beginning of each period, the agent receives an idiosyncratic preference shock. With probability α, he becomes a *Type*-1 agent, who consumes in the morning market and produces in the afternoon market. Similarly, with probability α, he becomes a *Type*-2 agent, who produces in the morning market and consumes in the afternoon market. The agent who consumes is called a buyer and the one who produces is called a seller. With probability $1 - 2\alpha$, the agent becomes neither Type-1 nor Type-2, and enters the night market directly.

In the morning market, the buyer obtains utility $u(q)$ from consuming q units of output. The cost function of the seller is q. We assume that there exists $q^* > 0$, such that $u'(q^*) = 1$. Similarly, in the afternoon market, the buyer obtains utility $\hat{u}(\hat{q})$ from consuming $\hat{q}$ units of output. The cost function of the seller is $\hat{q}$. We let $\hat{s}(q) = \hat{u}(q) - q$ denote the surplus. Let $\hat{q}^* > 0$ be such that $\hat{u}'(\hat{q}^*) = 1$. In the night market, each agent obtains utility $U(c)$ from consuming c units of general goods and disutility h from producing h units of goods. Let $c^* > 0$ be such that $U'(c^*) = 1$. The function u, $\hat{u}$ and U satisfy $u' > 0$, $\hat{u}' > 0$, $U' > 0$, $u'' < 0$, $\hat{u}'' < 0$, $U'' < 0$ and $u'(0) = \hat{u}'(0) = U'(0) = +\infty$.

The role of money is to facilitate trades in the morning and afternoon markets, where buyers and sellers are anonymous. The buyers use money to pay. Money is divisible and storable, but it is intrinsically useless. The growth rate of the money supply M is γ.

2.2. Night market

At night, the agent solves

$$W(m) = \max_{c,h}\{U(c) - h + \beta V_{+1}(m_{+1})\} \tag{1}$$

subject to the budget constraint $c = h + \phi(m + T - m_{+1})$, where $\beta > 0$ is a discount factor, ϕ is the price of money, $T = \gamma M$ is the lump-sum transfer, and $V(m)$ denotes the value function of the agents at the beginning of each period. The first-order conditions are $U'(c) = 1$ and

$$\phi = \beta V'_{+1}(m_{+1}). \tag{2}$$

Trades are efficient and the money balances at the beginning of each period are the same across agents. From the quasi-linearity of the utility function, $W(m) = \phi m + W(0)$.

2.3. Afternoon market

In the afternoon, the buyer (i.e., Type-2) makes a take-it-or-leave-it offer $(\hat{q}, \hat{d})$ to the seller (i.e., Type-1), where $\hat{q}$ is the quantity and $\hat{d}$ is the monetary transfer. The participation constraint of a seller is $W(m^s) \le -\hat{q} + W(m^s + \hat{d})$, which is simplified as $\phi\hat{d} - \hat{q} \ge 0$. The buyer solves

$$\hat{V}^b(m) = \max_{\hat{q},\hat{d}}\{\hat{u}(\hat{q}) + W(m - \hat{d})\} \quad \text{s.t } \hat{d} \le m \text{ and } \phi\hat{d} - \hat{q} \ge 0. \tag{3}$$

The buyer chooses the offer so that the participation constraint binds. Eq. (3) reduces to

$$\hat{V}^b(m) = \max_{\hat{d}:\hat{d}\le m}\{\hat{u}(\phi\hat{d}) - \phi\hat{d}\} + \phi m + W(0). \tag{4}$$

We let

$$\hat{v}(x) = \hat{u}(\min\{x, \hat{q}^*\}) - \min\{x, \hat{q}^*\} + x. \tag{5}$$

It satisfies $\hat{v}(x) = \hat{u}(x)$ if $x < \hat{q}^*$, and $\hat{v}(x) = \hat{s}(\hat{q}^*) + x$ if $x > \hat{q}^*$. Eq. (4) implies that $\hat{V}^b(m) = \hat{v}(\phi m) + W(0)$. Since the participation constraint is binding, the value function of the seller is $\hat{V}^s(m) = W(m)$.

2.4. Morning market

In the morning, the buyer (i.e., Type-1) makes a take-it-or-leave-it offer (q, d) to the seller (i.e., Type-2) where (q, d) is the terms of trade. The participation constraint of a seller who has m^s dollars is $\hat{V}^b(m^s+d) - q \ge \hat{V}^b(m^s)$. Table 1 summarizes the nominal balances of the agents at the end of each sub-period.

Let $V^b(m, m^s)$ denote the value function of the buyer who has m dollars when the seller holds m^s dollars. In the afternoon market, he becomes a seller with value function W. Since the participation constraint is binding, we obtain

$$V^b(m, m^s) = \max_{d:d\le m}\{u(\hat{V}^b(m^s + d) - \hat{V}^b(m^s)) - \phi d\} + \phi m + W(0). \tag{6}$$

As u and $\hat{V}$ are increasing and concave, the objective function is concave in d. Therefore, the choice of d is optimal if and only if it satisfies

$$\hat{V}^{b\prime}(m^s + d)u'(\hat{V}^b(m^s + d) - \hat{V}^b(m^s)) \ge \phi, \tag{7}$$

and the equality holds if $d < m$. The value function of the seller is $V^s(m) = \hat{V}^b(m)$.

As $V(m) = \alpha E_{m^s}[V^b(m, m^s)] + \alpha V^s(m) + (1 - 2\alpha)W(m)$, (2) implies that

$$i = \frac{\alpha}{\phi_{+1}}\left(E_{m^s}\left[\frac{\partial V^b_{+1}(m, m^s)}{\partial m}\right] + \frac{\partial V^s_{+1}(m)}{\partial m}\right) - 2\alpha, \tag{8}$$

where $i = \frac{\phi}{\beta\phi_{+1}} - 1$ is the nominal interest rate. In the steady state, $i = (1 + \gamma)/\beta - 1$. We let $z = \phi m$ be the real value of money balances. Similarly, we let $\rho = \phi d$ denote the real value of money transfer in the morning market.

3. Multiple equilibria

Here we discuss steady states.

3.1. Equilibrium condition

First, we assume that the constraint $d \le m$ binds in the morning. If $m^s = m$, from (6), $\frac{\partial V^b(m,m^s)}{\partial m} = \hat{V}^{b\prime}(2m)u'(\hat{V}^b(2m) - \hat{V}^b(m))$. As $\hat{V}^{b\prime}(m) = V^{s\prime}(m) = \phi\hat{v}'(\phi m)$, (8) reduces to

$$i = \alpha\{\hat{v}'(2z)u'(\hat{v}(2z) - \hat{v}(z)) + \hat{v}'(z) - 2\}. \tag{9}$$