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# Spectral and wavelet based assessment of congestive heart failure patients

Mehmet Engin*

*Electrical and Electronics Engineering Department, Faculty of Engineering, Ege University, Turkey*

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**Abstract**

Heart rate variability (HRV) can be quantified, among others, in the spectral and wavelet domain. The wavelet transform (WT) is an alternative method for the analysis of non-stationary signals. Some recent work shows that the scale-dependent WT standard deviation of the R–R intervals of human ECG can be used to distinguish patients with certain forms of cardiac pathological function from normal subjects. In this paper, we show an explicit relationship between variance of WT and corresponding spectral measure. Also, the statistics of the estimator for variance of WT is obtained. Numerical simulations support the theoretical results. By comparing expected value and variance and spectral measures, we conclude that WT measures are able to diagnose certain cardiac system function.


*Keywords:* Heart rate variability (HRV); Wavelet transform; Power spectral density (PSD); Statistical analysis

## 1. Introduction

Heart rate variability (HRV) is a recognized parameter for assessing autonomous nervous system activity. Adaptations of activity of the autonomous nervous system have been widely studied through HRV, which was first described by extracting the physiological rhythms embedded in its signal [1]. Indeed, physiological regulations, particularly blood pressure adaptations, pharmacological responses to many drugs, as well as numerous clinical applications, have been explored through its use [1]. This interest is notably reinforced by the known direct relationship between autonomous nervous system tone and heart rate, sinoatrial stretch, or myocardial contractility [1]. Different physiological parameters can be addressed, such as respiratory sinus arrhythmia mediated by parasympathetic activity, thermo regulatory fluctuations in vasomotor tone, and baroreflex control. There is a significant relationship between the autonomic nervous system and cardiovascular mortality, including sudden cardiac death. HRV is commonly used to describe variations of human heart interbeat intervals (R–R) measured using non-invasive electrocardiographic methods. While HRV analysis has been shown to be a useful clinical tool, its use as an early predictor of underlying pathology is still under investigation [2]. The interbeat interval time series of the human heart exhibits scaling behavior, as evidenced by the power-law form of its spectrum [3]. However, some features associated with physiological markers are also present in the power spectrum at particular frequencies. Moreover, it is well known that the heart beat time series is non-stationary, reflecting biological adaptability. Different mathematical methods have been used to analyze HRV. Among these, the Fourier transform is the one most commonly chosen but one that is, however, limited to stationary signals. When calculating a signal expansion, the primary concern is localization of a given basis function in time and frequency. For example, in Fourier transform, the functions used in the analysis are infinitely sharp in their frequency localization (they exist at one precise frequency) but have no time localization because of their infinite extend. There are various ways to define the localization of a particular basis function, but they are all related to the spread of the function in time and frequency [4]. To overcome this limitation, we applied the wavelet transform (WT), which offers two complementary interesting features [1]. One can define intervals $I_t$ and $I_w$ which estimates 90% of the energy of the time and frequency domain functions, respectively, and are centered on the center of gravity of $|f(t)|^2$ and $|F(w)|^2$. This defines what we call a tile in the

* Tel.: +90 232 3886024; fax: +90 232 3886024.
*E-mail address:* mehmet.engin@ege.edu.tr.

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frequency domain. For simplicity, we assumed a complex basis function. A real basis function would be represented by two mirror tiles at positive and negative frequencies. All elementary operations (e.g., shifting and scaling) conserve the surface of the time–frequency tile. In the scaling case, resolution in frequency was traded for resolution in time. The scaling is a fundamental operation used in the WT. Because of this, and assuming that a basis function is a bandpass filter as in wavelet analysis, high-frequency basis functions are obtained by going to small scales, and therefore, scale is loosely related to inverse frequency [4]. First, the WT allows a temporally localized sliding analysis of the signal, thus giving access at any instant to time status of the HRV, as, for example, when the balance of autonomous nervous system equilibrium is suddenly modified by acute clinical situations such as anesthesia or pharmacological interventions. Second, the shape of the WT differs from the fixed sinusoidal shape of the Fourier transform and can be designed to better fit the contours of the analyzed signal, allowing a better quantitative measurement. Multiresolution wavelet analysis provides an ideal means of decomposing non-stationary signals into its components at different scales. One of the main concepts of wavelet theory is the interpretation of WT in terms of multiresolution decomposition. This is especially useful for fast wavelet algorithms and for the initial approximations that usually have to be performed on the signal and wavelets. One of the main results of the multiresolution theory is the existence of a function $\psi(t)$—mother wavelet—constructed from the scaling function, and such that the set of $\psi(t-k)$ is an orthonormal basis. Power spectral based determination of HRV is now a routine tool for assessment of autonomic function. In recent studies, wavelet analysis of HRV was shown to discriminate between healthy subjects and those with some forms of underlying cardiac pathology [5]. The discriminating measurement was the standard deviation, $\sigma(m)$, which is the square root of variance; of the dyadic discrete wavelet transform (DWT) of the sequence of R–R heart beat intervals. However, for reliable clinical use, we must clearly understand the statistical properties of this discriminating measure. In this study, we used the properties of $\sigma(m)$ to demonstrate its link to power spectral density (PSD) measures of the same data. We assess their relative utility by evaluating the bias, variance, and frequency resolution of estimators both measures.

## 2. Background

### 2.1. Heart rate (HR) analysis

Spectral analysis of the rhythmic fluctuations in the beat-to-beat time series of electrocardiogram have led to the identification of three fairly distinct peaks: high (0.15–0.5 Hz), low (0.07–0.14 Hz), and very low (0.02–0.06 Hz) frequency bands [5]. HRV measures have been linked with manifold pathological and psychopathological conditions such as cardiovascular (CV) disease, diabetes, anxiety disorders, and attentional deficits. The two basic forms of HRV analysis are often designated as time and frequency domain measures. Time domain HRV indices are derived directly from interbeat intervals. Frequency domain measures such as spectral analysis mathematically decompose the HR time series into its component frequencies. One of the most frequently raised issues concerning spectral analysis on HRV data is the importance of stationarity in the time series [6]. In general, the stationarity of the signal refers to the invariance of its distributional characteristics over time. Most of the biomedical signals, including HR, have non-stationary properties. However, the multiresolution joint time-frequency analysis is suited for the examination of non-stationary signals such as HRV signals.

### 2.2. Properties of spectral and DWT variance measures

DWT was defined in the following way [5]:

$$W[m,n] = 2^{-m/2} \sum_{i=0}^{M-1} \tau(i)\psi(2^{-m}i - n), \tag{1}$$

where the scale variable $m$ and the translation variable $n$ are non-negative integers, $\tau(i)$ is the discrete time sequence of R–R intervals, $\psi$ is the wavelet basis function, and $M$ represents the total number of intervals. The variance of $W[m,n]$ as the function of $m$, which we will define as $D(m)$, is considered as the main parameter. In our work, we demonstrate the equivalence between $D(m)$ and PSD measures of the same data.

The analysis and interpretation is clearer in the continuous domain, but the results translate easily to the discrete domain. The continuous wavelet transform (CWT) of a signal $\tau(t)$, for an arbitrary wavelet function, is defined as [5]

$$\mathrm{CWT}^{\tau}(\alpha, b) = \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \tau(t)\psi^* \left[\frac{t-b}{\alpha}\right] \mathrm{d}t, \tag{2}$$

where $\alpha$ and $b$ are scale and translation parameters, respectively, $\psi$ is a wavelet basis function, and $*$ is complex conjugate. Since the expected value of CWT is 0, the variance of CWT at scale $\alpha$ is [5]

$$D(\alpha) = E\left[|\mathrm{CWT}^{\tau}(\alpha, b)|^2\right], \tag{3}$$

where $E$ is expectation operator. For a wide-sense stationary signal $\tau(t)$, this leads to

$$D(\alpha) = \alpha \int_{-\infty}^{\infty} R(\alpha y)\mathrm{CWT}^{\psi}(1, y)\,\mathrm{d}y \tag{4}$$

or alternatively

$$D(\alpha) = \alpha \int_{w=-\infty}^{\infty} \times S(w) \left[\int_{y=-\infty}^{\infty} \mathrm{CWT}^{\psi}(1, y)\exp(\mathrm{j}w\alpha y)\,\mathrm{d}y\right] \mathrm{d}w, \tag{5}$$

where $R(\alpha y)$ is the autocorrelation function of $\tau(t)$ and $\mathrm{CWT}^{\psi}(1, y)$ is the WT of the wavelet itself (termed the wavelet Kernel). $S(w)$ denotes the PSD of the signal.

Eq. (5) shows that the WT variance is directly related to the PSD through an integral transform. Hence, any discrimination seen in a $D(\alpha)$ based statistics such as $\sigma(m)$ should