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Asymmetric Nash bargaining solutions and competitive payoffs

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HIGHLIGHTS

- We link asymmetric Nash bargaining and competitive behavior in markets.
- Every bargaining game is a market game.
- Asymmetric Nash bargaining solutions correspond to competitive equilibrium payoffs.
- The bargaining weights are reflected in the market through the equilibrium prices.

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ABSTRACT

We establish a link between cooperative and competitive behavior. For every possible vector of weights of an asymmetric Nash bargaining solution there exists a market that has this asymmetric Nash bargaining solution as its unique competitive payoff vector.

1. Introduction

We bring together NTU bargaining games and competitive markets. By combining results from the literature we show how asymmetric Nash bargaining solutions can be justified in a general equilibrium framework as competitive payoff vectors of certain *markets*. Hereby, the term market refers to a class of economies used in the literature on market games with non-transferable utility, as in Billera and Bixby (1974) and Qin (1993).

The idea to justify Nash bargaining solutions by using economies is not new. Trockel (1996) already gives a direct interpretation of a bargaining game as an Arrow–Debreu economy with production and private ownership, a so-called bargaining economy. Similarly, Trockel (2005) uses large coalition production economies to establish a core equivalence of the Nash bargaining solution. By using the markets of Qin (1993) we establish a link between the literature on market games and the bargaining economies of Trockel (1996, 2005).

The analysis proceeds as follows: we first give an overview of the concepts that we use from cooperative game theory, including NTU bargaining games. Then we introduce markets and competitive payoffs. Afterward, we present the results and finally we conclude.

2. Cooperative games

We consider cooperative games with non-transferable utility (NTU). Let $N = \{1, \ldots, n\}$ with $n \in \mathbb{N}$ and $n \geq 2$ be the set of players. Let $\mathcal{N} = \{S \subseteq N | S \neq \emptyset\}$ be the set of non-empty coalitions.

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For a coalition $S \in \mathcal{N}$ define $\mathbb{R}^S = \{x \in \mathbb{R}^n | x_i = 0, \ \forall i \notin S\}$ and $\mathbb{R}^S_+ = \left\{x \in \mathbb{R}^n_+ | x_i = 0, \ \forall i \notin S\right\}$. A *compactly (and convexly) generated NTU game* is a pair (N, V), where the coalitional function V associates with each non-empty coalition $S \in \mathcal{N}$ a set of feasible payoff allocations, given by $V(S) = C^S - \mathbb{R}^S_+$ with $C^S \subseteq \mathbb{R}^S$ being non-empty, compact (and convex).

Let (N, V) be a compactly generated NTU game and let $\lambda \in \mathbb{R}^N_+$ be a vector of utility weights. Define the *λ-transfer game* of (N, V) by (N, V_λ) with $V_\lambda(S) = \{u \in \mathbb{R}^S | \lambda \cdot u \leq v_\lambda(S)\}$ where $v_\lambda(S) = \max\{\lambda \cdot u | u \in V(S)\}$.[1]
As for the λ-transfer game only payoff change rates λ_i/λ_j matter, we can assume without loss of generality that λ is normalized, i.e. $\lambda \in \Delta^n = \left\{\lambda \in \mathbb{R}^n_+ | \sum_{i=1}^n \lambda_i = 1\right\}$. Define the positive unit simplex by $\Delta^n_{++} = \{\lambda \in \mathbb{R}^n_{++} | \sum_{i=1}^n \lambda_i = 1\}$.

The *core* $C(V)$ of an NTU game (N, V) is defined as the set of those utility allocations that are achievable by the grand coalition N such that no coalition S can improve upon any of them. Formally, $C(V) = \{u \in V(N) | \forall S \in \mathcal{N} \, \forall u' \in V(S) \, \exists i \in S \text{ such that } u'_i \leq u_i\}$. A refinement of the core is the *inner core* $IC(V)$ of a compactly generated NTU game (N, V), defined as $IC(V) = \{u \in V(N) | \exists \lambda \in \Delta \text{ such that } u \in C(V_\lambda)\}$ where $C(V_\lambda)$ denotes the core of the λ-transfer game of (N, V). This means a vector u is in the inner core if and only if u is affordable by the grand coalition N and if u is in the core of an appropriately chosen λ-transfer game. If a utility allocation u is in the inner core, then u is also in the core. For compactly and convexly generated NTU games it is known that the vectors of supporting weights for a utility vector in the inner core must all be strictly positive (Qin, 1993, Remark 1, p. 337).

We consider a special class of NTU games, namely NTU bargaining games. Let $B \subseteq \mathbb{R}^n$ be a compact, convex set and assume that there exists at least one $b \in B$ with $b \gg 0$.[2] Define an *NTU bargaining game*[3] (N, V) with the generating set B (with the properties as above) using the player set N and the coalitional function V defined by

$$V(\{i\}) := \left\{b \in \mathbb{R}^n | b_i \leq 0, b_j = 0, \ \forall j \neq i\right\} = \{0\} - \mathbb{R}^{\{i\}}_+,$$

$$V(S) := \{0\} - \mathbb{R}^S_+ \quad \text{for all } S \text{ with } 1 < |S| < n,$$

$$V(N) := \left\{b \in \mathbb{R}^n | \exists b' \in B : b \leq b'\right\} = B - \mathbb{R}^n_+.$$

The definition of an NTU bargaining game reflects the idea that smaller coalitions than the grand coalition do not gain from cooperation. They cannot reach higher utility levels than the singleton coalitions simultaneously for all its members. Only in the grand coalition can every individual be made better off. The *asymmetric Nash bargaining solution with a vector of weights* $\theta = (\theta_1, \ldots, \theta_n) \in \Delta^n_{++}$, θ-asymmetric for short, for an NTU bargaining game (N, V) is defined as the maximizer of the θ-asymmetric Nash product given by $\prod_{i=1}^n u_i^{\theta_i}$ over the set $V(N)$.

Hereby, we consider the symmetric Nash bargaining solution as one particular asymmetric Nash bargaining solution, namely the one with the vector of weights $\theta = \left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$. Hence, the correct interpretation of "asymmetric" in this sense is "not necessarily symmetric".[4] As the NTU bargaining game (N, V) is compactly and convexly generated, the set $V(N)$ is closed and convex and hence the maximizer above exists.

3. Markets

A *market*[5] is given by $\mathcal{E} = \left(X^i, Y^i, \omega^i, u^i\right)_{i \in N}$ where for every individual $i \in N$: $X^i \subseteq \mathbb{R}^\ell_+$ is a non-empty, closed and convex set, the consumption set, with $\ell \geq 1$ the number of commodities; $Y^i \subseteq \mathbb{R}^\ell$ is a non-empty, closed and convex set, the production set, such that $Y^i \cap \mathbb{R}^\ell_+ = \{0\}$; $\omega^i \in X^i - Y^i$, the initial endowment vector; and $u^i : X^i \to \mathbb{R}$ is a continuous and concave function, the utility function. Let $S \in \mathcal{N}$ be a coalition. An S-allocation is a tuple $\left(x^i\right)_{i \in S}$ such that $x^i \in X^i$ for each $i \in S$. The feasible S-allocations are the allocations that the coalition S can achieve by redistributing their initial endowments and by using the production possibilities within the coalition, formally given by

$$F(S) = \left\{(x^i)_{i \in S} | x^i \in X^i \text{ for all } i \in S, \sum_{i \in S}(x^i - \omega^i) \in \sum_{i \in S} Y^i\right\}.$$

An NTU game that is representable by a market is an *NTU market game*, which means that there exists a market $\mathcal{E} = (X^i, Y^i, \omega^i, u^i)_{i \in N}$ such that $(N, V_{\mathcal{E}}) = (N, V)$ with

$$V_{\mathcal{E}}(S) = \{u \in \mathbb{R}^S | \exists (x^i)_{i \in S} \in F(S), u_i \leq u^i(x^i), \ \forall i \in S\}.$$

For an NTU market game there exists a market such that the set of utility allocations that a coalition can reach according to the coalitional function coincides with the set of utility allocations that are generated by feasible S-allocations in the market or that give less utility than some feasible S-allocation.

A *competitive equilibrium* for a market $\mathcal{E}$ is a tuple $\left((\hat{x}^i)_{i \in N}, (\hat{y}^i)_{i \in N}, \hat{p}\right) \in \mathbb{R}^{\ell n}_+ \times \mathbb{R}^{\ell n}_+ \times \mathbb{R}^\ell_+$ such that

(i) $\sum_{i \in N} \hat{x}^i = \sum_{i \in N}(\hat{y}^i + \omega^i)$ (market clearing),
(ii) for all $i \in N$, $\hat{y}^i$ solves $\max_{y^i \in Y^i} \hat{p} \cdot y^i$ (profit maximization),
(iii) and for all $i \in N$, $\hat{x}^i$ is maximal with respect to the utility function u^i in the budget set $\{x^i \in X^i | \hat{p} \cdot x^i \leq \hat{p} \cdot (\omega^i + \hat{y}^i)\}$ (utility maximization).

Given a competitive equilibrium $\left((\hat{x}^i)_{i \in N}, (\hat{y}^i)_{i \in N}, \hat{p}\right)$, its *competitive payoff vector* is defined as $\left(u^i\left(\hat{x}^i\right)\right)_{i \in N}$.

4. Results

We investigate the relationship between asymmetric Nash bargaining solutions and competitive payoffs of a market that represents an NTU bargaining game. From Billera and Bixby (1974, Theorem 2.1) it is known that the class of compactly and convexly generated NTU market games coincides with the class of totally balanced games. The total balancedness of NTU bargaining games can easily be verified. Therefore, first note the following proposition.

Proposition 1 (*Billera and Bixby, 1973, Theorem 4.1*)**.** *Every NTU bargaining game is a market game.*

1 Qin (1994, p. 433) gives the following interpretation of the λ-transfer game ($\lambda \gg 0$): "The idea of the λ-transfer game may be captured by thinking of each player as representing a different country. The utilities are measured in different currencies, and the ratios λ_i/λ_j are the exchange rates between the currencies of i and j".

2 For normalization purposes we assume here that the disagreement outcome is 0 and that $B \subseteq \mathbb{R}^n_+$. Nevertheless, the results presented here can easily be generalized to the case that the disagreement point is not equal to 0.

3 See for example Billera and Bixby (1973, Section 4).

4 Similarly to the symmetric Nash bargaining solution, the asymmetric Nash bargaining solution satisfies the axioms Invariance to Affine Linear Transformations, Pareto Optimality and Independence of Irrelevant Alternatives. As shown in Roth (1979, p. 20), for example, these axioms together with an appropriate Asymmetry assumption fixing the vector of weights characterize an asymmetric Nash bargaining solution.

5 This particular type of economies was considered in Billera (1974), Qin (1993), Qin and Shubik (2009), among others.