Economics Letters 121 (2013) 298–301

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Marginal effects of a bivariate binary choice model

Takuya Hasebe *

Ph.D. Program in Economics, Graduate Center, City University of New York, 365 Fifth Avenue, New York, NY, USA

HIGHLIGHTS

- The marginal effects of the copula-based bivariate binary choice model are derived.
- The signs of the marginal effects are shown to be determined by the signs of the coefficients using the properties of a copula.
- A real-data application is provided.

ARTICLE INFO

Article history:
Received 1 August 2013
Received in revised form
26 August 2013
Accepted 28 August 2013
Available online 4 September 2013

JEL classification:
C31
C35

Keywords:
Copula
Binary choice
Marginal effect

ABSTRACT

This paper discusses the copula-based approach of a bivariate binary choice model. We derive the marginal effects of explanatory variables on an outcome of interest (both direct and indirect) in the model. We also show that the signs of the marginal effects are determined by the signs of the coefficient parameters. A real-data application is provided.

1. Introduction

A bivariate binary choice model is a frequently used model in applied microeconomics. In particular, its recursive form is practically useful when estimating the effect of an endogenous dummy (treatment) variable on a binary outcome. In binary choice models (or any other nonlinear model), the coefficients on explanatory variables do not measure the marginal effects of these variables. In order to interpret the economic significance of explanatory variables, it is necessary to compute the marginal effects from the estimated coefficients. In this paper, we derive the marginal effects in a bivariate binary choice model. Besides a traditional bivariate probit approach, we also consider a copula-based approach. We illustrate by considering a real-data application.

2. The model

The model consists of two equations: for an observation i, $i = 1, \ldots, N$,

$$\begin{cases} y_{1i} = 1(\alpha y_{2i} + x'_{1i}\beta + \varepsilon_{1i} \geq 0), \\ y_{2i} = 1(x'_{2i}\beta_2 + \varepsilon_{2i} \geq 0), \end{cases} \tag{1}$$

where $1(\cdot)$ is an indicator function. The vectors x_1 and x_2 are sets of explanatory variables.[1] The variable y_2 in the first equation is the primary interest, but it is potentially endogenous. The endogeneity issue arises when the errors ε_1 and ε_2 are not independent. In order to estimate the coefficient parameters consistently, the dependence between ε_1 and ε_2 needs to be taken into consideration. The almost exclusively used approach is maximum likelihood estimation (MLE), where the log likelihood function has a general form $\ln L = \sum_{i=1}^{N} \ln \Pr(y_{1i}, y_{2i})$, where $\Pr(y_{1i}, y_{2i})$ is the joint probability of y_1 and y_2.

In order to implement MLE, we need to specify the joint distribution of the error terms, $F(\varepsilon_1, \varepsilon_2)$. A standard approach is to assume that these errors are jointly normally distributed. Under bivariate normality, $F(\varepsilon_1, \varepsilon_2) = \Phi_2(\varepsilon_1, \varepsilon_2; \rho)$, where $\Phi_2(\cdot)$ is the cumulative distribution function (cdf) of the bivariate normal distribution with the coefficient of correlation ρ. The joint probability of y_1 and y_2 is $\Pr(y_1, y_2) = \Phi_2\left(s_1(\alpha y_2 + x'_1\beta_1), s_2(x'_2\beta_2); s_1 s_2\rho\right)$, where $s_j = 2y_j - 1$ for $j = 1, 2$. The model is called a (recursive) bivariate probit model (Greene, 2008).

* Tel.: +1 9175838359.
E-mail address: thasebe@gc.cuny.edu.

[1] Wilde (2000) shows that the model is identified even when the sets x_1 and x_2 are the same.

0165-1765/$ – see front matter
http://dx.doi.org/10.1016/j.econlet.2013.08.028

3. The copula approach

Winkelmann (2012) discusses the copula-based approach to allow non-normal dependence. In short, a copula $C(\cdot)$ is a function that binds univariate marginal distributions together to represent a joint distribution. Using a copula, the joint distribution of ε_1 and ε_2 is $F(\varepsilon_1, \varepsilon_2) = C(F_1(\varepsilon_1), F(\varepsilon_2); \theta)$, where $F_j(\cdot)$ is the cdf of a univariate distribution of ε_j and θ is a dependence parameter that governs the degree of dependence. For notational simplicity, we suppress the dependence parameter from the expression hereafter. We also assume that ε_j is a continuous random variable, although the copula approach is not limited to the continuous case. Define $u_j = F(\varepsilon_j)$ with $u_j \in [0, 1]$; then, copula function C satisfies the following properties: (i) $C(1, u_2) = u_2$ and $C(u_1, 1) = u_1$, (ii) $C(0, u_2) = C(u_1, 0) = 0$, and (iii) for $u_1 \le u_1'$ and $u_2 \le u_2'$, $C(u_1', u_2') - C(u_1, u_2') - C(u_1', u_2) + C(u_1, u_2) \ge 0$. The last property says that C is 2-increasing.

These properties are essential to determine the signs of the marginal effects of the copula-based bivariate binary choice model. Define $C_j(u_1, u_2) = \partial C(u_1, u_2)/\partial u_j$ for $j = 1, 2$ and $C_{12}(u_1, u_2) = \partial^2 C(u_1, u_2)/\partial u_1 \partial u_2$. Then, it can easily be seen that property (iii) implies that $C_{12}(u_1, u_2) \ge 0$. It can also be shown that $C_j(u_1, u_2) \ge 0$ and $1 - C_j(u_1, u_2) \ge 0$. To show the former, set $u_2 = 0 < u_2'$. Then, by properties (ii) and (iii), $C(u_1', u_2') - C(u_1, u_2') \ge 0$, which is directly followed by $C_1(u_1, u_2) \ge 0$. To show the latter, set $u_2 < u_2' = 1$. Then, by properties (i) and (iii), $C(u_1', 1) - C(u_1, 1) - C(u_1', u_2) + C(u_1, u_2) = u_1' - u_1 - \left(C(u_1', 1) - C(u_1, 1)\right) \ge 0$. Then, it is clear that $1 - C_1(u_1, u_2) \ge 0$.

Using a copula, the joint probability of y_1 and y_2 is

$$\Pr(y_1, y_2) = y_1 y_2 - y_1 s_2 F_2(-x_2'\beta_2) - y_2 s_1 F_1(-\alpha y_2 - x_1'\beta_1) + s_1 s_2 C(F_1(-\alpha y_2 - x_1'\beta_1), F_2(-x_2'\beta_2)).$$

In applications, researchers need to choose a copula from several available copulas.[2] Researchers also have freedom to choose the marginal distributions $F_1(\cdot)$ and $F_2(\cdot)$. When the marginal distributions are standard normal distributions and the copula is Gaussian, the copula-based model is the same as the bivariate probit model.

4. Marginal effects

In the binary choice model, the coefficient parameters do not have direct economic interpretations. We need to compute the marginal effects of the explanatory variables. The outcome of the structural equation, y_1, is usually the main interest. Researchers are often interested in the effect of the endogenous dummy y_2 on the expected value of y_1. It can be computed as $(1 - F_1(-\alpha - x_1'\beta)) - (1 - F_1(-x_1'\beta_1)) = F_1(-x_1'\beta) - F_1(-\alpha - x_1'\beta)$. Since $F_1(\cdot)$ is a non-decreasing function, the effect has the same sign as α.

The effects of other explanatory variables on y_1 may also be interesting. Depending whether the variable appears in x_1 or in x_2, the channels through which the explanatory variable affects y_1 differ. While a change in x_1 directly affects $E(y_1)$ (direct effect), a change in x_2 affects $E(y_1)$ through a change in y_2 (indirect effect). Even though Christofides et al. (1997) derive the marginal effects in the bivariate probit model, the model is not in recursive form. Greene (1998) derives the marginal effects in recursive form. In this paper, we derive the marginal effects of the copula-based model, and we also show that the signs of the marginal effects are determined by the signs of the coefficient parameters.

In a general expression, $E(y_1) = \Pr(y_1 = 1) = \Pr(y_1 = 1, y_2 = 0) + \Pr(y_1 = 1, y_2 = 1)$. The marginal effects are obtained by taking derivatives with respect to the corresponding variables.[3]

Under normality, $E(y_1) = \Phi_2(x_1'\beta_1, -x_2'\beta_2; -\rho) + \Phi_2(\alpha + x_1'\beta_1, x_2'\beta_2; \rho)$. The direct effect is its derivative with respect to x_1:

$$\frac{\partial E(y_1)}{\partial x_1} = \left[\phi(x_1'\beta_1) \times \Phi\left(\frac{-x_2'\beta_2 + \rho x_1'\beta_1}{\sqrt{1-\rho^2}}\right) + \phi(\alpha + x_1'\beta_1) \times \Phi\left(\frac{-x_2'\beta_2 + \rho(\alpha + x_1'\beta_1)}{\sqrt{1-\rho^2}}\right)\right] \times \beta_1.$$

The sign of this marginal effect is the same as the sign of β_1, since all the terms in the square brackets are positive. The indirect effect is the derivative with respect to x_2:

$$\frac{\partial E(y_1)}{\partial x_2} = \phi(x_2'\beta_2) \times \left[\Phi\left(\frac{\alpha + x_1'\beta_1 - \rho x_2'\beta_2}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_1'\beta_1 - \rho x_2'\beta_2}{\sqrt{1-\rho^2}}\right)\right] \times \beta_2.$$

The sign of this effect depends on the signs of β_2 and α. When $\alpha > 0$, the square bracket expression will be positive, since $\Phi(\cdot)$ is an increasing function, and then, the effect has the same sign as β_2. When $\alpha < 0$, the sign of the effect is opposite to the sign of β_2 as the square bracket expression becomes negative.

In the copula-based model, $E(y_1) = 1 - F_1(-\alpha - x_1'\beta_1) + C(F_1(-\alpha - x_1'\beta_1), F_2(-x_2'\beta_2)) - C(F_1(-x_1'\beta_1), F_2(-x_2'\beta_2))$. Taking the derivative, the direct effect is

$$\frac{\partial E(y_1)}{\partial x_1} = \left\{\left[1 - C_1(F_1(-\alpha - x_1'\beta_1), F_2(-x_2'\beta_2))\right] \times f_1(\alpha + x_1'\beta_1) + C_1(F_1(-x_1'\beta_1), F_2(-x_2'\beta_2)) \times f_1(x_1'\beta_1)\right\} \times \beta_1,$$

where $f_1(\cdot)$ is the probability distribution function (pdf) of ε_1 with $f_1(\cdot) \ge 0$. As shown above, $C_1(\cdot)$ and $1 - C_1(\cdot)$ are positive. Therefore, the sign of the direct effect is the same as that of β_1. The indirect effect is

$$\frac{\partial E(y_1)}{\partial x_2} = -\left[C_2(F_1(-\alpha - x_1'\beta_1), F_2(-x_2'\beta_2)) - C_2(F_1(-x_1'\beta_1), F_2(-x_2'\beta_2))\right] \times f_2(-x_2'\beta_2) \times \beta_2.$$

As in the bivariate probit model, the indirect effect in the copula-based model also depends on the signs of α and β_2. Suppose that $\alpha > 0$. Then, $F_1(-\alpha - x_1'\beta_1) \le F_1(-x_1'\beta_1)$ since the cdf is non-decreasing. Given this, the fact that $C_{12}(\cdot) \ge 0$ implies that the square bracket expression is negative, and the effect has the same sign as that of β_2. Likewise, if $\alpha < 0$, then the effect has the opposite sign to that of β_2.

These discussions are the marginal effects of continuous variables so that we are able to take the derivatives. For discrete variables, we can compute the differences, $E(y_1|x_1 = 1) - E(y_1|x_1 = 0)$ and $E(y_1|x_2 = 1) - E(y_1|x_2 = 0)$, as the direct effect and the indirect effect, respectively. In the same way as for continuous variables, the signs of the effects of discrete variables can be derived from the properties of a copula.

2 The list of copulas is available, for example, in Nelsen (2006).

3 More precisely, $E(y_1) = \int E(y_1|x)dF(x) \approx N^{-1}\sum_i^N E(y_1|x_i)$, where x is a set of variables, x_1 and x_2, and $F(X)$ is the distribution function of x. The following derivations of the marginal effects are the derivatives of $E(y_1|x_i)$. The marginal effect on $E(y_1)$ can be computed by summing over observations. This marginal effect is referred to as "average marginal effect". For notational simplicity, we suppress the summation.