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A strict ex-post incentive compatible mechanism for interdependent valuations

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HIGHLIGHTS

- This paper addresses mechanism design in an interdependent value setting.
- This problem is important and non-trivial because of an impossibility result.
- The proposed mechanism improves the classic mechanism given by Mezzetti (2004).
- The improvement is in making the second round strictly incentive compatible.
- However, the improvement comes at the cost of subgame perfection.

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ABSTRACT

The impossibility result by Jehiel and Moldovanu says that in a setting with interdependent valuations, any efficient and ex-post incentive compatible mechanism must be a constant mechanism. Mezzetti circumvents this problem by designing a two stage mechanism where the decision of allocation and payment are split over the two stages. This mechanism is elegant, however it has a major weakness. In the second stage, agents are weakly indifferent about reporting their valuations truthfully: an agent's payment is independent of her reported valuation and truth-telling for this stage is by assumption. We propose a modified mechanism which makes truthful reporting in the second stage a strict equilibrium.

1. Introduction

In the classical independent private values model (Mas-Colell et al., 1995), each agent observes her valuation which depends on the allocation and her own private type. One can design efficient, dominant strategy incentive compatible mechanisms in this setting, e.g., the VCG mechanism achieves these properties. However, in many real world scenarios where agents collaboratively solve a problem, e.g., joint projects in multinational companies, or large distributed online projects, such as *crowdsourcing* experiments like the DARPA red balloon challenge (Pickard et al., 2011), the valuation of an agent depends on the private types of other agents as well. The model where the valuation of an agent for an allocation depends not only on her private type but also on the types of other agents, is the *interdependent value* model and has also received a great deal of attention in the literature (Krishna, 2009).

The interdependent value model poses a more difficult challenge for mechanism design. In increasing generality, Maskin (1992), Jehiel and Moldovanu (2001), and Jehiel et al. (2006) have shown that the efficient social choice function cannot *generically* be ex-post implemented. Ex-post implementation requires agents to be truthful about their own type reports when all others are reporting their types truthfully. This is a strong negative result — it rules out the existence of a mechanism that takes type reports from the agents and yields an allocation and a payment rule which satisfies ex-post incentive compatibility and efficiency. However, Mezzetti (2004) has shown that these goals can be achieved if the mechanism designer can split the allocation and payment decisions into two stages. The agents report their types in Stage 1, and the designer implements an allocation based on that. Then, each

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agent observes her own valuation and reports the values to the designer in Stage 2. The designer then proposes a payment based on the two-stage reports. This mechanism is called *generalized Groves mechanism* and in the Nash equilibrium the allocation is efficient. However, a drawback of the mechanism pointed out by Mezzetti (2004) is that the agents are indifferent between truth-telling and lying in Stage 2. Hereafter, we will refer to this mechanism as the *classic mechanism*.

In this paper, we propose a mechanism called Value Consistent Pivotal Mechanism (VCPM) that overcomes this difficulty. In particular it proposes a different set of payments from the classic mechanism in Stage 2 which makes it a strict ex-post Nash equilibrium for each agent to reveal her valuation truthfully at this stage.

The question may arise whether this mechanism hurts some other properties of the classic mechanism. Since VCPM yields the same payoff to the agents as that of the classic in equilibrium, it continues to satisfy all the properties that the classic mechanism satisfies in equilibrium. For example, we show that in a restricted problem domain, a refinement of our mechanism satisfies *individual rationality* (IR) as it is satisfied by the classic mechanism. However, truth-telling in the classic mechanism is also a subgame perfect equilibrium. On the contrary, truth-telling need not be a subgame perfect equilibrium in VCPM. We illustrate this with an example.

The rest of the paper is organized as follows. In Section 2, we introduce the model and define certain properties. In Section 3, we present the mechanism and discuss its properties. We conclude the paper in Section 4.

2. Model and definitions

Let the set of agents be denoted by $N = \{1, \dots, n\}$. Each agent observes her private type $\theta_i \in \Theta_i$. Let $\Theta = \times_{i\in N} \Theta_i$ denote the type profile space where $\theta \equiv (\theta_1, \dots, \theta_n)$ be an element of Θ. We will denote the type profile of all agents except agent i by $\theta_{-i} \equiv (\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_n) \in \Theta_{-i}$. Types are drawn independently across agents and each agent can only observe her own type and does not know the types of other agents. We consider a standard quasi-linear model. Therefore, the payoff u_i of agent i is the sum of her value v_i and transfer p_i.

We will consider a two stage mechanism similar to that of Mezzetti (2004), because of the impossibility result by Jehiel and Moldovanu (2001). We call this mechanism Value Consistent Pivotal Mechanism (VCPM).

In Stage 1, agents are asked to report their types, and after that the mechanism designer chooses an alternative from the set A via the allocation function $a : \Theta \to A$. We denote the reported types by $\hat{\theta}$, hence, the allocation for such a report is given by $a(\hat{\theta})$.

All agents then experience the consequence of the allocation via the valuation function which is defined by $v_i : A \times \Theta \to \mathbb{R}$, for all $i \in N$. Note: the value function is different from the independent private value setting, where it is a mapping $v_i : A \times \Theta_i \to \mathbb{R}$. This difference makes mechanism design in interdependent value settings difficult as discussed in Section 1.

In Stage 2, agents report their experienced valuations; transfers given by $p_i : \Theta \times \mathbb{R}^n \to \mathbb{R}$, $\forall i \in N$ are then decided by the designer. If the reported valuations are $\hat{v} \in \mathbb{R}^n$, the transfer to agent i is given by $p_i(\hat{\theta}, \hat{v})$.

The two stage mechanism VCPM is graphically illustrated in Fig. 1.

2.1. Definitions

As discussed earlier, in the paper we will consider only two stage mechanisms, where in Stage 1, agents report their types and in Stage 2, their experienced valuations. The allocation decision is made after Stage 1 and the payment after Stage 2. It is, therefore, necessary to define the notions of efficiency, truthfulness, and voluntary participation, in this setting.

We consider only quasi-linear domains where the payoff is the sum of the valuation and transfer. A mechanism M in this domain is fully characterized by a tuple of allocation and payment $\langle a, p\rangle$. For a truthful mechanism in this setting, we need to ensure that it is truthful in both stages. In Stage 1, truthfulness implies that the agents report their true types. In the second round, the valuation is a function of the allocation chosen in Stage 1. Here truthfulness would mean that they report their observed valuations due to that allocation.

The true type profile is given by θ. With a slight abuse of notation, we represent the true valuation vector by $v = (v_1(a(\theta), \theta), \dots, v_n(a(\theta), \theta))$ under mechanism $M = \langle a, p\rangle$. Let us denote the payoff of agent i by $u_i^M(\hat{\theta}, \hat{v}|\theta, v)$ under mechanism M when the reported type and value vectors are $\hat{\theta}$ and $\hat{v}$ respectively, while the true type and value vectors are given by θ and v. Therefore, due to the quasi-linear assumption, the payoff is given by,

$$u_i^M(\hat{\theta}, \hat{v}|\theta, v) = v_i(a(\hat{\theta}), \theta) + p_i(\hat{\theta}, \hat{v}).$$

Definition 1 (*Efficiency (EFF)*). A mechanism $M = \langle a, p\rangle$ is efficient if the allocation rule maximizes the sum valuation of the agents. That is, for all θ,

$$a(\theta) \in \operatorname*{argmax}_{a\in A} \sum_{j\in N} v_j(a, \theta).$$

Definition 2 (*Ex-post Incentive Compatibility (EPIC)*). A mechanism $M = \langle a, p\rangle$ is ex-post incentive compatible if reporting the true type and valuation is an ex-post Nash equilibrium of the induced game. That is, for all true type profiles $\theta = (\theta_i, \theta_{-i})$ and true valuation profiles $v = (v_i, v_{-i}) = (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta))$, and for all $i \in N$,

$$u_i^M((\theta_i, \theta_{-i}), (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta))|\theta, v)$$
$$\geq u_i^M((\hat{\theta}_i, \theta_{-i}), (\hat{v}_i, v_{-i}(a(\hat{\theta}_i, \theta_{-i}), \theta))|\theta, v), \quad \forall \hat{\theta}_i, \hat{v}_i.$$

Definition 3 (*Ex-post Individual Rationality (EPIR)*). A mechanism $M = \langle a, p\rangle$ is ex-post individually rational if the payoff of each agent in the true type and valuation profile is non-negative. That is, for all $i \in N$, $\theta = (\theta_i, \theta_{-i})$, and $v = (v_i, v_{-i}) = (v_i(a(\theta), \theta), v_{-i}(a(\theta), \theta))$,

$$u_i^M(\theta, v|\theta, v) \geq 0.$$

Subset allocation (SA). Later in this paper, we will focus on a problem domain named *subset allocation*, where the allocation set is the set of all subsets of the agents, i.e., $A = 2^N$. In such a setting, we assume that the valuation of agent i is given by,

$$v_i(a, \theta) = \begin{cases} v_i(a, \theta_a) & \text{if } i \in a, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

We use θ_a to denote the type vector of the allocated agents, i.e., $\theta_a = (\theta_j)_{j\in a}$, $\forall a \in A$. This means that when agent i is not selected, her valuation is zero, and when she is selected, the valuation depends only on the types of the selected agents. This restricted domain is relevant for distributed projects in organizations, where the skill level of only the allocated agents matter in the value achieved by the other allocated agents. The skill levels of all the workers/employees participating in the project impact the success or failure of the project, and the reward or loss is shared by the participants of the project. This and several other examples of collaborative task execution falls under the SA domain, which makes it interesting to study.

3. Main results

With the dynamics of the mechanisms as in Fig. 1, the mechanism design problem is to design the allocation and the transfer rules. In VCPM, we adopt the following allocation and transfer rules.