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An alternative interpretation of random noise in rank-order tournaments

Andrei Barbos *

Department of Economics, University of South Florida, Tampa, FL, United States

HIGHLIGHTS

- We define a variant of the canonical model of contests in Lazear and Rosen (1981).
- This variant proposes an alternative interpretation of the random additive noise.
- Players observe their final outputs at the time when they choose the inputs.
- This allows employing the model in situations where outputs are observed.

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ABSTRACT

We propose an alternative interpretation of additive random noise from the canonical framework of Lazear and Rosen (1981) amenable to applications where players do observe their final outputs at the time when inputs are chosen. This interpretation is suitable to model situations where participants do not know the precise distribution of the population from which their opponents are drawn. The variant of the model that we define is strategically equivalent to that in Lazear and Rosen (1981).

1. Introduction

Situations where the allocation of resources results from competition among agents who choose levels of costly inputs, and have their payoffs determined by the rank order of performance rather than by its absolute value, are commonly referred to in the economic literature as tournaments, or contests. Examples include patent races, sporting events, elections, sales contests, promotional competitions, education filters, etc.

The seminal paper of Lazear and Rosen (1981) (henceforth LR) introduced a canonical framework in which to analyze the so-called difference-form contests where individuals' inputs are only observed by the contest organizer with some random additive noise. A typical example, also employed in LR, is that of workers who exert effort which cannot be directly measured by their employer, but only imperfectly inferred from that worker's output, which may depend not only on the worker's effort but also on luck or on his ability. The additive noise modeling choice employed in this framework has two merits. First, it captures situations where players' inputs are indeed only imperfectly observable by the contest organizer, and thus cannot be contracted upon. Second, from a formal viewpoint, it ensures the existence of a *pure* strategy Nash equilibrium amenable to a sharper analysis.

An implicit assumption underlying the additive noise information structure employed in these difference-form models is that participants in the contest do not observe the precise values of the outputs that they produce *at the time when they make their decisions*. In other words, a player does not observe the noise that affects his input choice. While this assumption is valid in many situations, in a variety of others it is not. For instance, workers usually exert effort over a long period of time, and thus are able to adjust the intensity of their effort depending on how previous effort translated into output. One can think of such a case, then, as one where the worker actually chooses the output level rather than the input. In the standard model of LR, this would lead to a different analysis and insights.

In this short article, we propose an alternative interpretation of the additive random noise from the difference-form contests

* Correspondence to: 4202 East Fowler Ave, CMC 206, Tampa, FL 33620-5500, United States. Tel.: +1 813 974 6514; fax: +1 813 974 6510.
E-mail address: andreibarbos@gmail.com.
URL: http://sites.google.com/site/andreibarbos/.

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which allows employing this framework to model situations where players do observe their final outputs when they choose their inputs, i.e., when they essentially compete by choosing outputs. In particular, we define a slightly modified version of the LR model which has the property that it induces the same family of best response functions, and the same equilibrium strategies and outcomes. As suggested, for instance, by Chowdhury and Sheremeta (2012) this property means that the two types of contests are essentially strategically equivalent.

In the application of the labor tournament analyzed in LR, the key specification of our variant of the model is that each player in the contest knows how the abilities of his opponents are distributed around a certain mean, but does not have any *prior* information on what that mean ability is. This model is thus particularly suitable to characterize contests where players do not know precisely the distribution of the population from which the participants in the contest are drawn. A situation where this is satisfied is when individuals are unfamiliar with their opponents, such as, when they are selected by the tournament organizer from an extended pool of possible participants.

More generally, the situations where our model applies are those where participants enjoy head starts, where, following Siegel (forthcoming), a head start is defined as an advantage that a player has at the outset of a competition. For instance, in a patent race, firms usually differ with respect to the time when they engage in a particular line of research, and therefore while each firm knows the stage of their research, they may not know how advanced it is relative to their competitors.[1] Similarly, when applying for a particular job opportunity an individual may not know the quality of applicants he is competing against.

In the main section of the article we present the LR model and our variant, as well as two propositions that state the condition under which the two models are strategically equivalent. Their proofs are relegated to the Appendix.

2. Results

There are n heterogeneous players who compete in a contest. Each player i has an *initial score* a_i, which is a random variable. A player's choice is his *input* in the contest, denoted by $e_i \in \mathbb{R}_+$. It costs $c(e_i)$ to provide input at the level e_i, where c satisfies $c(0) = 0, c' > 0$ and $c'' > 0$. The *total score* of player i is $a_i + e_i$. Players do not observe the inputs or scores of their opponents at the time when they make their decisions. The contest organizer observes the total scores, but cannot disentangle the two components, which renders it infeasible to write contracts contingent on players' inputs. Instead, the contract specifies a payment scheme where the player with the kth highest total score earns a payoff $w^{(k)}$, with $w^{(1)} > w^{(2)} > \cdots > w^{(n)}$. To simplify exposition, we first present the analysis for the simpler case, where $n = 2$, also considered in LR.

In the LR framework, a_i is interpreted as noise that affects player i's input, and which is not observed by the player at the time when he has to choose e_i. Assuming that the random variables a_i are independent and have continuous probability density functions $f_i(\cdot)$ with mean equal to 0, player i's expected payoff under an arbitrary strategy profile (e_i, e_{-i}) is

$$w^{(2)} + (w^{(1)} - w^{(2)}) \Pr(e_i + a_i > e_{-i} + a_{-i}) - c(e_i). \tag{1}$$

Considering that the second order condition is satisfied, player i's best response function $e_i(e_{-i})$ is then given implicitly by the equation

$$\left(w^{(1)} - w^{(2)}\right) \int_{-\infty}^{\infty} f_j(e_i - e_{-i} + t) f_i(t) dt - c'(e_i) = 0. \tag{2}$$

Next, we present our modified framework. As announced earlier, we allow players to observe their initial scores at the time when they choose their inputs, which implies that they essentially observe their final outputs $a_i + e_i$. Player i's initial score (or head start), a_i, is the sum between a common component, b_0, and an idiosyncratic random component, b_i, of mean 0. Our key modeling assumption is that b_0 is a *random* variable, whose realization is not observed by the agent at the time when he chooses his input. Thus, each player i observes his initial score a_i, but not the components b_0 or b_i. The random variables b_0, b_1 and b_2 are independent. The density function of b_0 is denoted by $\varphi_{b_0}(\cdot)$, and the densities of b_i by $f_i(\cdot)$.

The following proposition identifies the condition on b_0 under which the best response functions of the two players from this modified model are the same as the best response functions from the LR model. The functional form equivalence of the equations that define implicitly these best response functions is up to the distributions of the idiosyncratic randomness from each model.[2]

Proposition 1. *In a contest with 2 players, the two versions of the model generate the same families of best response functions if and only if* $\varphi_{b_0}(\overline{b}_0) = \varphi_{b_0}(\overline{b}_0')$ *for any* $\overline{b}_0, \overline{b}_0' \in \mathbb{R}$.

The condition elicited in Proposition 1 requires that b_0 have a non-standard uniform distribution on the real line. This improper distribution has a natural interpretation as a way to characterize a situation where an agent has no prior information on a variable.[3] This suggests that the variant of the additive noise framework that we propose is particularly applicable to model situations where players compete in contests where they are sufficiently unfamiliar with their opponents. Players do learn from prior experience that participants in any contest are selected so as to have a certain distribution of initial scores around a particular mean but do not know what the mean initial score is in any given contest. Conditional on competing in a contest, a player's knowledge of his own initial score is informative of the mean initial score of the players in that contest, but Proposition 1 states that if the agent has no *prior* information on this mean to correlate his private information with, then the resulting contest is strategically equivalent to the contest defined in LR.

Our proposed alternative interpretation can be employed in more general models of random additive noise developed on the LR framework. To evaluate the robustness of the result of Proposition 1 to more general modeling specifications, we also study here the case of an arbitrary number of players n. To simplify exposition, we assume that in the two versions of the model, the probability density functions $f_i(\cdot)$ are identical, and denote it by $f(\cdot)$. We demonstrate the strategic equivalence of the two specifications of the contest by showing that they induce the same (symmetric) equilibrium strategies, rather than the complete families of best response functions. This allows forgoing the relatively cumbersome computation of best response functions to non-symmetric profiles of opponents' strategies.

[1] In equilibrium, a firm may know the resources spent by other firms in the industry, and thus be able to infer how much time it would take them to develop a patent, but if head starts are private information, the firm will also not know precisely when its competitors will file their patent applications.

[2] This means that two equations that define the best responses in the two models have the same functional form for given distributions $f_i(\cdot)$ of the random variables a_i in the LR model, and b_i in our model.

[3] For instance, in the statistics literature, this improper distribution is employed when the analyst has no or little prior information, but highly informative data. To avoid employing an improper distribution, some authors also define a uniform distribution on a bounded interval to model lack of information. The issue with this approach is that it does impose a certain degree of informativeness of the prior, as it implies that the likelihood that the random variable takes a particular value on one side of the boundary is infinitely higher than the likelihood that it takes some value from the other side.