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A simple spatial dependence test robust to local and distributional misspecifications☆

Ying Fang[a], Sung Y. Park[b,*], Jinfeng Zhang[c]

[a] The Wang Yanan Institute for Studies in Economics, MOE Key Laboratory of Econometrics, Fujian Key Laboratory of Statistic Science, Xiamen University, Xiamen, Fujian 361005, China

[b] School of Economics, Chung-Ang University, 84 Heuksok-Ro, Dongjak-Gu, Seoul, Republic of Korea

[c] The Center for Economic Research, Shandong University, Jinan, Shandong 250100, China

HIGHLIGHTS

- We propose a test for spatial dependence which is robust to local misspecification and distributional misspecification.
- We find that Burridge (1980)'s test is robust to distributional misspecification.
- We find that Anselin, Bera, Florax and Yoon (1996)'s test is robust to distributional misspecification.

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ABSTRACT

In this paper, we derive a class of modified score tests robust to local and distributional misspecifications for testing spatial error autocorrelation and spatial lag dependence. The proposed tests are general enough to include several popular tests for the spatial dependence as special cases. Moreover, we show that the popular test statistics proposed by Burridge (1980) and Anselin et al. (1996) are robust to distributional misspecification although they are derived under normality assumption.

1. Introduction

Moran (1950) proposes a seminal test, Moran's I-test, for spatial autocorrelation in a regression model framework, but the test does not provide the nature of the spatial process that causes spatial autocorrelation, particularly, whether the spatial dependence is due to an autoregressive error process or omitted spatially lagged dependent variables. Burridge (1980) extends Moran's I-test based on the Lagrange multiplier (LM) principle to test the spatial error autocorrelation without considering the presence of spatially lagged dependent variables, and furthermore, Anselin (1988b) proposes a LM test for the spatial error autocorrelation in the case of the presence of the spatially lagged dependent variable. However, the test involves a nonlinear optimization or applies a numerical search technique. Anselin et al. (1996) propose a modified score test for the spatial error autocorrelation in the presence of local misspecification to the parameter corresponding to the spatial lag dependence. Comparing to the Ansenlin's LM test, the latter only requires the ordinary least squares (OLS) residuals under the null hypothesis and has little computational burden (Bera and Bilias, 2001). Yet one potential problem of these tests is that the underlying probability density may not be correctly specified, i.e., there may exist the distributional misspecification problem.

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* Corresponding author. Tel.: +82 2 820 5622; fax: +82 2 812 9718.
E-mail address: sungpark@cau.ac.kr (S.Y. Park).

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We develop a modified score test for spatial dependence, robust to both the local and distributional misspecifications. Local parametric misspecification arises when some nuisance parameters deviate locally from their true values. Distributional misspecification occurs when the underlying data generating process (DGP) is not correctly specified. When nuisance parameters are locally deviated from their true values, the score statistic has a non-zero drift term in general (Davidson and MacKinnon, 1987; Saikkonen, 1989). Bera and Yoon (1993) propose a modified score test robust to the local misspecification. On the other hand, when the underlying probability distribution is misspecified, some standard results are not valid any more. For example, the information matrix (IM) equality is invalid under distributional misspecification. Making inferences with the distributional misspecification can cause size distortion of the test statistics. For example, White (1982) suggests a modified LM test based on the restricted quasi-maximum likelihood (QML) estimator to deal with the distributional misspecification. Bera et al. (2007) propose a score test that is not only robust to local misspecification but also to distributional misspecification in the spirit of White (1982) and Bera and Yoon (1993). Recently, Baltagi and Yang (2013a,b) modify the standard LM test for spatial error dependence robust against distributional misspecification for cross-sectional and panel data models, respectively. In the spatial econometrics literature, the corresponding robust score (LM) test which deals with the distributional misspecification or both the local and distributional misspecifications have not been studied yet. There are some studies that suggest some other types of robust tests. However, their test statistics are based on other different estimation methods, and neither of them considers the local misspecification.[1]

A modified test can be constructed by properly adjusting the mean and variance of the usual score test statistics, and therefore, it has correct asymptotic size. The test statistics can be simplified when either the error term follows the normal distribution or the nuisance parameters are estimated consistently. In the next section, we first review spatial dependence tests in a spatial autoregressive model with a spatial autoregressive disturbance. We then develop new score tests robust to both local parametric and distributional misspecifications. The paper concludes in Section 3.

2. A modified score test robust to local and distributional misspecifications

Consider the following spatial autoregressive model with a spatial autoregressive disturbance (SARAR) model (Anselin, 1988a; Case, 1991; Anselin et al., 1996; Kelejian and Prucha, 2010; among others):

$$y = \rho W_1 y + X\beta + \epsilon, \tag{2.1}$$

$$\epsilon = \lambda W_2 \epsilon + u, \tag{2.2}$$

where y is the dependent variable, X is a $N \times k$ matrix of explanatory variables, β denotes a $k \times 1$ unknown parameter vector, ρ and λ are scalar spatial parameters, W_1 and W_2 are $N \times N$ known spatial weight matrices, ϵ is an $N \times 1$ vector of regression disturbances, and u is an $N \times 1$ vector of innovations with $u_i \sim i.i.d(0, \sigma^2)$ for $i = 1, 2, \ldots, N$.

For the notational convenience, we denote $\theta = (\beta', \sigma^2, \lambda, \rho)'$, $\gamma = (\beta', \sigma^2, \lambda)'$, $\eta = (\beta', \sigma^2)'$, $A = I_N - \rho W_1$, $B = I_N - \lambda W_2$, $G_A = W_1 A^{-1}$ and $G_B = W_2 B^{-1}$. Under the normality assumption, following Anselin (1988a), the log-likelihood function of (2.1) and (2.2) is given by

$$\ln L(\theta) = -\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\sigma^2 + \ln|A| + \ln|B| - \frac{1}{2\sigma^2}u'u, \tag{2.3}$$

where $u = B(Ay - X\beta)$.

The test statistics previously proposed by Burridge (1980), Anselin (1988b), and Anselin et al. (1996) are derived under the normality assumption of the error terms. However, when the distribution is misspecified, all tests are generally invalid and may have the size distortion problem. Let $g(y)$ and $f(y, \theta)$ be the true model and the specified model, respectively. When $g(y) \neq f(y, \theta)$, the information matrix $K(\theta)$ and the negative expected Hessian matrix $J(\theta)$ are not equivalent any more; see Appendix A for details. Hence, the variance–covariance matrix of the score statistic should be modified. In general, not only the mean but also the variance of the score test statistic have to be adjusted accordingly to take care of the local and distributional misspecifications.

2.1. A modified score test for spatial error dependence

For testing the spatial error dependence, the null hypothesis of interest is $H_0^\lambda : \lambda = 0$. Burridge (1980) proposes a one-directional score test for H_0^λ by assuming $\rho = 0$ in (2.1):

$$RS_\lambda = \frac{\left(\tilde{u}'W_2\tilde{u}/\tilde{\sigma}^2\right)^2}{T_{22}}, \tag{2.4}$$

where $T_{22} = \mathrm{tr}\left[(W_2 + W_2')W_2\right]$, $\tilde{u} = y - X\tilde{\beta}$ and $\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/N$. Here $\tilde{\theta} = (\tilde{\beta}', \tilde{\sigma}^2, 0, 0)'$ denotes the constraint maximum likelihood estimator (MLE) under H_0^λ. The above statistic RS_λ converges in distribution to χ_1^2 under H_0^λ. However, if the nuisance parameter $\rho \neq 0$, for example, which is contaminated by a local deviation such that $\rho = \delta_1/\sqrt{N}$ where δ_1 is a nonzero finite constant, the Burridge statistic converges to a noncentral chi-square distribution, which implies that RS_λ generally over-reject the null hypothesis even if H_0^λ is true. Anselin et al. (1996) construct a robust score test for the spatial error autocorrelation by eliminating the noncentral term. Using the one-step method-of-scoring estimator, the modified score test is given by

$$RS_\lambda^L = \frac{\left[\tilde{u}'W_2\tilde{u}/\tilde{\sigma}^2 - T_{21}\left(N\tilde{J}_{\rho\cdot\eta}\right)^{-1}\tilde{u}'W_1 y/\tilde{\sigma}^2\right]^2}{T_{22} - T_{21}^2\left(N\tilde{J}_{\rho\cdot\eta}\right)^{-1}}, \tag{2.5}$$

where $\tilde{u}$ are the OLS residuals, $\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/N$, $T_{21} = \mathrm{tr}\left[(W_2 + W_2')W_1\right]$, $\tilde{J}_{\rho\cdot\eta} = \frac{1}{N\tilde{\sigma}^2}\left[T_{11}\tilde{\sigma}^2 + \left(W_1X\tilde{\beta}\right)' M_X\left(W_1X\tilde{\beta}\right)\right]$ for $T_{11} = \mathrm{tr}\left[(W_1 + W_1')W_1\right]$ and $M_X = I_N - X\left(X'X\right)^{-1}X$. RS_λ^L converges to χ_1^2 distribution under H_0^λ even though ρ deviates locally from 0 such that $\rho = \delta_1/\sqrt{N}$.

We consider a modified score test robust to both local and distributional misspecifications, i.e., $g(y) \neq f(y, \theta)$ and $\rho = \rho_0 + \delta_1/\sqrt{N}$ where ρ_0 is a finite constant and $\delta_1 > 0$. Under $H_0^\lambda : \lambda = 0$, the modified score test is given by

$$RS_\lambda^{LD} = \frac{\left\{\tilde{u}'W_2\tilde{u}/\tilde{\sigma}^2 - \tilde{T}_{2A}\left(N\tilde{J}_{\rho\cdot\eta}\right)^{-1}\left[-\mathrm{tr}(G_A) + \tilde{u}'W_1 y/\tilde{\sigma}^2\right]\right\}^2}{T_{22} - \tilde{T}_{2A}^2\left(N\tilde{J}_{\rho\cdot\eta}\right)^{-1} + \frac{1}{N}\tilde{T}_{2A}^2\left(\tilde{J}_{\rho\cdot\eta}^{-2}\tilde{B}_{\rho\cdot\eta}^{*}\right)}, \tag{2.6}$$

where the OLS residual $\tilde{u} = y - \rho_0 W_1 y - X\tilde{\beta}$, $\tilde{\sigma}^2 = \tilde{u}'\tilde{u}/N$ and $\tilde{T}_{2A} = \mathrm{tr}\left[(W_2' + W_2)G_A\right]$, $\tilde{J}_{\rho\cdot\eta} = \frac{1}{N}[\tilde{T}_{AA} - \frac{2}{N}tr^2(G_A) + \frac{1}{\sigma^2}(G_AX\tilde{\beta})'$

[1] An incomplete list includes, Anselin (1990), Anselin and Kelejian (1997), Kelejian and Robinson (1998), Anselin and Moreno (2003), Saavedra (2003) and Yang (2010) among others.