



On optimal head starts in all-pay auctions



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HIGHLIGHTS

- We consider a two-player all-pay auction in which the designer chooses a head start.
- The designer maximizes a convex combination of the expected total and highest effort.
- Unless the weight on the highest effort is 1, small head starts are worse than none.
- A positive head start is optimal iff the weight on the highest effort is large enough.

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ABSTRACT

We consider a two-player all-pay auction with symmetric independent private values that are uniformly distributed. The designer chooses the size of a head start that is given to one of the players. The designer's objective is to maximize a convex combination of the expected highest effort and the expected aggregate effort. Unless the weight on the highest effort is one, small head starts are always worse than no head start. Moreover, the optimal head start is strictly positive if and only if the weight on the highest effort is large enough.

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1. Introduction

Contests are among the most frequently used mechanisms with applications in job promotion, procurement, competitions for grants or college admission, innovation, R&D, sports and so forth. The organizer of a contest often directly or indirectly benefits from the effort that participants invest. Hence, natural objectives when designing a contest are to maximize the expected aggregate effort or the expected highest effort. The latter objective is, for instance, particularly relevant in innovation contests where the organizer plans to implement only the best innovation, e.g., the best of the prototypes or algorithms that are submitted by the participants. Another example is a firm that adopts the most creative advertising campaign out of several proposals.

Surprisingly, the contest literature has mainly focused on the problem of maximizing aggregate effort. A common finding is that if contestants are asymmetric in their abilities, it is beneficial for the designer to bias the contest in favor of weaker participants: leveling the playing field enhances competition and thereby elicits more effort. However, if contestants are symmetric, the designer typically prefers an unbiased contest; see, e.g., Lazear and Rosen (1981), Clark and Riis (2000), Kirkegaard (2012), and Franke et al. (2014).

In this paper, we examine whether this conclusion remains to hold if the designer is – either exclusively or at least partially – interested in maximizing the expected highest effort. We consider an all-pay auction with two ex ante symmetric players that are privately informed about their uniformly distributed valuations. The designer can choose the size of a *head start* (Konrad, 2002; Seel, 2014; Siegel, 2014) that is granted to one of the players. This player wins whenever the sum of her effort and the head start is higher than the other player's effort. Whereas any nonzero head start reduces the expected aggregate effort in our setting, we show that the optimal head start is strictly positive if the expected highest

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effort is sufficiently important to the designer. Hence, the common intuition of leveling the playing field does not necessarily apply.

Two recent papers show that unequal treatment of ex ante symmetric players can be beneficial if the designer is not or not only interested in the efforts of the contestants. Kawamura and Moreno de Barreda (2014) find that a head start may increase the probability of selecting the highest ability player in an all-pay auction that is played under complete information. Pérez-Castrillo and Wettstein (2013) study innovation contests where the quality of a player's innovation is a function of his privately known type and his effort. They show that the designer may profit from discriminating among contestants by letting the size of the prize vary with the winner's identity.

2. Model and preliminaries

There are two players $i = 1, 2$ competing for a single prize. Each player i privately knows his valuation for the prize v_i , which is a realization of the random variable V_i . The random variables V_1 and V_2 are identically and independently distributed according to the continuous cumulative distribution function F . They have a strictly positive density f on their support $[0, \bar{v}]$. The virtual valuation function

$$\psi(v) := v - \frac{1 - F(v)}{f(v)} \quad \text{is strictly increasing in } v. \quad (1)$$

Players compete in a first-price all-pay auction. Player 1 enjoys a head start $a \in [0, \bar{v}]$,¹ i.e., he wins if $b_1 + a > b_2$ and player 2 wins if $b_1 + a \leq b_2$, where b_i denotes the bid (invested effort) of player i . The (ex post) payoff of player i is $v_i - b_i$ if he wins and $-b_i$ otherwise. A pure strategy for player i is a function $\beta_i: [0, \bar{v}] \rightarrow \mathbb{R}_+$ so that player i bids $\beta_i(v_i)$ if his valuation is v_i .

From Kirkegaard (2012) it follows that in the Bayesian Nash equilibrium of this game for a given head start a , players use the equilibrium strategies

$$\beta_1^a(v) := \begin{cases} 0 & \text{if } v < \hat{v}, \\ \int_{\hat{v}}^v x dF(x) & \text{if } v \geq \hat{v}, \end{cases}$$

$$\beta_2^a(v) := \begin{cases} 0 & \text{if } v < \hat{v}, \\ a + \int_{\hat{v}}^v x dF(x) & \text{if } v \geq \hat{v}, \end{cases}$$

where \hat{v} is implicitly defined by $a = \hat{v}F(\hat{v})$.

Note that for $a > 0$, $0 = \beta_1^a(v) = \beta_2^a(v) < \beta_1^0(v) = \beta_2^0(v)$ for $v < \hat{v}$ whereas $0 < \beta_1^a(v) < \beta_1^0(v) = \beta_2^0(v) < \beta_2^a(v)$ for $v > \hat{v}$. A head start for player 1 induces both players to exert zero effort for low valuations. For high valuations, the head start leads to player 2 bidding more aggressively and player 1 bidding less aggressively compared to the benchmark without a head start.

The contest designer's objective is to maximize a weighted sum of the expected highest effort and the expected average effort²:

$$W_\gamma(a) := \gamma E[\max\{\beta_1^a(V_1), \beta_2^a(V_2)\}] + (1 - \gamma)E\left[\frac{1}{2}(\beta_1^a(V_1) + \beta_2^a(V_2))\right]$$

¹ We exclude head starts $a > \bar{v}$ from the analysis to avoid the trivial equilibrium in which both players always bid zero.

² Note that maximizing W_γ is equivalent to maximizing the weighted sum of the expected highest effort (weight γ') and the expected total effort (weight $1 - \gamma'$) for $\gamma = \frac{\gamma'}{2 - \gamma'}$.

where $\gamma \in [0, 1]$. We denote optimal head starts by $a^*(\gamma) := \arg \max_a W_\gamma(a)$.

First, consider $\gamma = 0$ such that the objective is the expected average effort

$$W_0(a) = \frac{1}{2} \hat{v} F(\hat{v})(1 - F(\hat{v})) + \int_{\hat{v}}^{\bar{v}} \int_{\hat{v}}^v x dF(x) dF(v). \quad (2)$$

Taking the derivative with respect to a , we find

$$W'_0(a) = \frac{d\hat{v}}{da} \left(-\frac{1}{2} \hat{v}(1 - F(\hat{v}))f(\hat{v}) - \frac{1}{2} \psi(\hat{v})F(\hat{v})f(\hat{v}) \right) = \frac{d\hat{v}}{da} \frac{1}{2} \int_0^{\hat{v}} (\psi(x) - \psi(\hat{v}))f(x)f(\hat{v})dx,$$

where we have used that $\int \psi(x)f(x)dx = -x(1 - F(x))$. Since $a = \hat{v}F(\hat{v})$, $\frac{d\hat{v}}{da} > 0$. Together with assumption (1), this implies $W'_0(a) < 0$ for all $a > 0$. Hence, if $\gamma = 0$, it is not beneficial to introduce a head start, i.e., $a^*(0) = 0$.³

Now, consider $\gamma = 1$ such that the objective is the expected highest effort. To determine $W_1(a)$ from the equilibrium strategies, observe that from an ex ante perspective, player i 's bid b_i is distributed according to $F((\beta_i^a)^{-1}(b_i))$. Let $X_a := \max\{\beta_1^a(V_1), \beta_2^a(V_2)\}$ be the highest bid and let G_a denote the cumulative distribution function of X_a with support $[0, \beta_2^a(\bar{v})]$. If $0 \leq a \leq \beta_1^a(\bar{v})$,

$$G_a(x) = \begin{cases} F(\hat{v})^2 & \text{if } x = 0, \\ F((\beta_1^a)^{-1}(x))F(\hat{v}) & \text{if } x \in (0, a), \\ F((\beta_1^a)^{-1}(x))F((\beta_2^a)^{-1}(x)) & \text{if } x \in [a, \beta_1^a(\bar{v})], \\ F((\beta_2^a)^{-1}(x)) & \text{if } x \in [\beta_1^a(\bar{v}), \beta_2^a(\bar{v})], \end{cases} \quad (3)$$

whereas if $\beta_1^a(\bar{v}) < a < \bar{v}$,

$$G_a(x) = \begin{cases} F(\hat{v})^2 & \text{if } x = 0, \\ F((\beta_1^a)^{-1}(x))F(\hat{v}) & \text{if } x \in (0, \beta_1^a(\bar{v})), \\ F(\hat{v}) & \text{if } x \in [\beta_1^a(\bar{v}), a], \\ F((\beta_2^a)^{-1}(x)) & \text{if } x \in [a, \beta_2^a(\bar{v})]. \end{cases} \quad (4)$$

The expected highest effort is given by

$$W_1(a) = \int_0^{\beta_2^a(\bar{v})} x dG_a(x) = \beta_2^a(\bar{v}) - \int_0^{\beta_2^a(\bar{v})} G_a(x) dx. \quad (5)$$

This expression is significantly less tractable than the expression for the average effort in (2). For our analysis of optimal head starts for $\gamma > 0$, we will therefore impose a specific distributional assumption.

3. The uniform case

We henceforth assume, in accordance with (1), valuations to be uniformly distributed on $[0, 1]$, i.e., $F(v) = v$. This implies $\hat{v} = \sqrt{a}$ and $\beta_1^a(v) = \beta_2^a(v) - a = \frac{1}{2}v^2 - \frac{1}{2}a$ for $v > \hat{v}$. In the following, we first consider the problem of maximizing the expected highest effort $W_1(a)$. Then we turn to studying optimal head starts that maximize the general objective function $W_\gamma(a)$ with $\gamma \in [0, 1]$.

³ For (irregular) type distributions that violate assumption (1), Kirkegaard (2012) finds that a small head start can be beneficial.

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