Economics Letters 124 (2014) 227–230

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Income stratification and between-group inequality

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HIGHLIGHTS

- I generalize a result on the decomposition of the Gini index to more than two groups.
- It is shown explicitly how overlapping of groups impacts between-group inequality.
- An overall index of income stratification is identified for the population.
- I tabulate the pairwise contributions of regions to global income stratification.

ARTICLE INFO

Article history:
Received 4 December 2013
Received in revised form
16 May 2014
Accepted 24 May 2014
Available online 2 June 2014

JEL classification:
D31
D63

Keywords:
Gini coefficient
Subgroup decomposability
Income stratification

ABSTRACT

The paper shows explicitly how the overlapping of groups impacts between-group inequality by generalizing a result on the group-wise decomposition of the Gini index to more than two groups. It is demonstrated that the ratio of Yitzhaki's measure of between-group inequality to the conventional measure is in general equal to one minus twice the weighted average probability that a random member of a richer (on average) group is poorer than a random member of a poorer (on average) group, and may therefore be interpreted as an overall index of income stratification in the population. The results are used to tabulate the contribution of each pair of regions in the world to the overall level of global income stratification.

1. Introduction

It is well known that the standard decomposition of the Gini index G by population groups does not yield an exact partition into between-group and within-group components, G_B and G_W respectively, unless the income ranges of the groups are non-overlapping (e.g., Mookherjee and Shorrocks, 1982). This has led both to an extensive literature exploring the nature of the "residual" from the standard decomposition (e.g. Lambert and Aronson, 1993; Lambert and Decoster, 2005) and to a parallel search for alternative decompositions that might prove more amenable to analysis and interpretation. In the latter vein, Yitzhaki and Lerman (1991) provides a partition of the Gini into between-group, within-group and overlapping components – G_b, G_w and G_o respectively – where overlapping is considered as the inverse of the sociological concept of 'stratification'. Yitzhaki (1994) subsequently combines the latter two elements into a single within-group measure G_{wo} that is explicitly written as a function of the degree of inequality within groups and the degree of overlapping between each pair of groups, but G_b is also affected by overlapping and it remains to be shown how this measure relates to the conventional between-group index G_B if there are more than two groups.[1]

2. Group-wise decomposition of the Gini index

Consider a population divided into $K \geq 2$ mutually exclusive and exhaustive groups that are ordered by mean income from the poorest to the richest group. Let Y_k, $F_k(Y_k)$, μ_k, p_k and q_k represent respectively the income (or some other relevant aspect of wellbeing) variable, cumulative distribution function, expected value, population share and income share of group k. The overall population $Y_u = Y_1 \cup Y_2 \cdots \cup Y_K$ is the union of all groups with distribution function $F_u(Y_u) = \sum_k p_k F_k(Y_k)$ and expected value $\mu_u = \sum_k p_k \mu_k$. The (fractional) ranking of group k incomes in

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[1] See Yitzhaki and Schechtman (2013) for a recent monograph on the Gini methodology.

http://dx.doi.org/10.1016/j.econlet.2014.05.025

the group l and overall income distributions are given as $F_l(Y_k)$ and $F_u(Y_k)$ respectively, with corresponding mean ranks $\bar{F}_{kl}$ and $\bar{F}_{ku}$.

Following Mookherjee and Shorrocks (1982), the conventional group-wise decomposition of the Gini index may be written as $G = 2\text{cov}(Y_u, F_u(Y_u))/\mu_u = G_B + G_W + R$ where $G_B = \sum_k \sum_l p_k p_l |\mu_l - \mu_k| / 2\mu_u$; $G_W = \sum_k p_k q_k G_k$ with $G_k = 2\text{cov}(Y_k, F_k(Y_k))/\mu_k$ denoting the Gini index of group k; and the residual R is interpreted as an 'interaction effect'. The alternative approach of Yitzhaki (1994) yields the exact decomposition $G = G_b + G_{wo}$ where $G_b = 2\sum_k p_k(\mu_k - \mu_u)\left(\bar{F}_{ku} - 0.5\right)/\mu_u$; and $G_{wo} = \sum_k q_k G_k O_k$ with O_k denoting the overlapping index of group k with the entire population. In turn $O_k = \sum_l p_l O_{lk}$ where the pair-wise overlapping index $O_{lk} = \text{cov}(Y_k, F_l(Y_k))/\text{cov}(Y_k, F_k(Y_k))$ lies in the open interval [0, 2] and is an increasing function of the fraction of group l that is located in the income range of group k, taking a value of zero when there is no overlap between the two groups and of one if the income distributions of the two groups are identical, i.e. $F_l(Y_k) = F_k(Y_k)$.

Thus $G_{wo} = G_W$ if there is perfect stratification in the sense of Lasswell (1965), since $O_{kk} = 1$ by definition, whereas $G_{wo} > G_W$ if the income ranges of the various groups overlap to any extent with the difference $R_W = G_{wo} - G_W$ given as:

$$\begin{aligned} R_W &= \sum_k q_k G_k \left(\sum_{l\neq k} p_l O_{lk}\right) \\ &= 2\sum_k p_k \left(\sum_{l\neq k} p_l \text{cov}(Y_k, F_l(Y_k))\right)/\mu_u \geqslant 0. \end{aligned} \tag{1}$$

Yitzhaki and Lerman (1991, p. 323) conclude that "inequality and stratification are inversely related", arguing that this relationship is consistent with relative deprivation theory in that "stratified societies can tolerate higher inequality than unstratified societies" since "As people become more (less) engaged with each other, they have less (more) tolerance for a given level of inequality". However, as Monti and Santori (2011) observe, this conclusion ignores the effect of overlapping on the between-group component G_b, which will also affect the overall level of inequality perceived by the society.

Yitzhaki and Lerman (1991, p. 322) note that $G_b = G_B$ if there is no overlapping and $G_b < G_B$ otherwise. Monti and Santori (2011) further demonstrate in the two group case that the ratio of G_b to G_B is equal to:

$$I = G_b/G_B = 1 - 2\text{Prob}(Y_1 > Y_2) \tag{2}$$

where $\text{Prob}(Y_1 > Y_2)$ is the probability of transvariation, i.e. the probability that the income of a random member of the poorer (on average) group is more than that of a random member of the richer (on average) group. To extend this result to the general case of $K \geq 2$ groups, note that G_b may also be expressed as:

$$\begin{aligned} G_b &= 2\sum_k p_k\mu_k \left(\sum_{l\neq k} p_l\left(\bar{F}_{kl} - 0.5\right)\right)/\mu_u \\ &= \sum_k \sum_{l>k} (p_k + p_l)\frac{(p_k\mu_k + p_l\mu_l)}{\mu_u} \\ &\quad \times \left(\frac{2\left(p_k\mu_k p_l\left(\bar{F}_{kl} - 0.5\right) + p_l\mu_l p_k\left(\bar{F}_{lk} - 0.5\right)\right)}{(p_k + p_l)(p_k\mu_k + p_l\mu_l)}\right) \\ &= \sum_k \sum_{l>k} (p_k + p_l)(q_k + q_l) G_b^{kl} \end{aligned} \tag{3}$$

where the first line follows since $\bar{F}_{ku} = \sum_l p_l \bar{F}_{kl}$ and $\bar{F}_{kk} = 0.5$, while G_b^{kl} denotes the Yitzhaki (1994) between-group index in the sub-population consisting only of groups k and l. Similarly, G_B can be written as:

$$\begin{aligned} G_B &= \sum_k \sum_{l>k} p_k p_l (\mu_l - \mu_k)/\mu_u \\ &= \sum_k \sum_{l>k} (p_k + p_l)(q_k + q_l) G_B^{kl} \end{aligned} \tag{4}$$

where G_B^{kl} denotes the between-group Gini in the sub-population consisting of groups k and l only. Using (2) and (4), (3) may be re-written as:

$$\begin{aligned} G_b &= \sum_k \sum_{l>k} (p_k + p_l)(q_k + q_l) G_B^{kl}\left(\frac{G_b^{kl}}{G_B^{kl}}\right) \\ &= \sum_k \sum_{l>k} p_k p_l (\mu_l - \mu_k)\{1 - 2\text{Prob}(Y_k > Y_l)\}/\mu_u \end{aligned} \tag{5}$$

from which it follows immediately that I will in general be equal to:

$$\begin{aligned} I = G_b/G_B &= \sum_k \sum_{l>k} w_{kl}(1 - 2\text{Prob}(Y_k > Y_l)) \\ &= 1 - 2\sum_k \sum_{l>k} w_{kl}\,\text{Prob}(Y_k > Y_l) \\ &= \sum_k \left\{\sum_{l<k} w_{kl}(0.5 - (1 - \text{Prob}(Y_k < Y_l)))\right. \\ &\quad \left. + \sum_{l>k} w_{kl}(0.5 - (\text{Prob}(Y_k > Y_l)))\right\} \end{aligned} \tag{6}$$

where $w_{kl} = p_k p_l(\mu_l - \mu_k)/\left(\sum_k \sum_{l>k} p_k p_l(\mu_l - \mu_k)\right) \geqslant 0$, with $\sum_k \sum_{l>k} w_{kl} = 1$ by definition, and the final line holds since $\text{Prob}(Y_k > Y_l) = (1 - \text{Prob}(Y_k < Y_l))$.

Hence I is in general equal to one less twice the weighted average probability of transvariation between the various pairs of groups in the population. In his study of earnings differentials Gastwirth (1975) proposes $TPROB = 2\text{Prob}(Y_1 > Y_2)$ as an index of overlapping between two groups, taking an "ideal" value of one when the two distributions are identical since $\text{Prob}(Y_1 > Y_2) = 0.5$ in this case. Thus I in (2) may be interpreted as the complementary index of non-overlapping or stratification, with (6) providing a generalization to two or more groups. I is a unit-free index that will take a maximum value of one when there is no overlap between any of the groups such that $\text{Prob}(Y_k > Y_l) = 0\,\forall k,\ l > k$; and will equal zero when the income distributions of all the groups are identical.[2] For $K > 2$, the extent to which non-overlapping between any pair of groups contributes to the overall level of stratification is an increasing function of their population shares and the difference in mean incomes between them. I is invariant to both the scaling and translation of incomes. It is also invariant to replication both of the population within existing groups and of groups.

I has previously been identified by Milanovic and Yitzhaki (2002, p. 161) "as an index indicating the loss of between group inequality due to overlapping". The difference $R_B = G_b - G_B$ can be written from (6) as:

$$\begin{aligned} R_B &= -2G_B \sum_k \sum_{l>k} w_{kl}\text{Prob}(Y_k > Y_l) \\ &= -2\sum_k \sum_{l>k} p_k p_l (\mu_l - \mu_k)\,\text{Prob}(Y_k > Y_l)/\mu_u \leq 0 \end{aligned} \tag{7}$$

[2] Negative values of I are also possible when mean incomes by group are negatively correlated with mean ranks.