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A Walrasian Rubinstein and Wolinsky model

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HIGHLIGHTS

- We revisit Rubinstein and Wolinsky (1985) matching and bargaining model.
- Our innovation is to make the flows unbalanced, rather than the stocks.
- We present a necessary and sufficient condition for the limit price to be Walrasian.
- The condition is the alignment of the initial buyer and seller stocks with the flows.

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ABSTRACT

We provide a full dynamic analysis of a continuous-time variant of Rubinstein and Wolinsky (1985) matching and bargaining model with unbalanced flows of buyers and sellers. The focus is on the price limit as the frictions of search are removed. It is found that a necessary and sufficient condition for the limit price to be Walrasian at all times is the alignment of the initial buyer and seller stocks with the flows.

1. Introduction

Since the influential articles by Rubinstein and Wolinsky (1985; RW hereafter) and Gale (1987), there is still no consensus among economists whether or not dynamic matching and bargaining markets become Walrasian in the frictionless limit. Most of the literature has focused on steady states, with the notable exceptions of Blouin and Serrano (2001) and Manea (2013).[1] This leaves open the important question as to how long it will take to reach the steady state, and whether the prices are close to Walrasian even away from the steady state.

In this paper, we investigate a continuous-time variant of the RW model as in Mortensen and Wright (2002), and provide a full dynamic analysis. In our model, buyers and sellers arrive over time are matched pairwise according to a constant returns to scale matching technology, and make take-it or leave-it offers to each other. Successful traders leave the market. We also allow for exogenous exit from the market, which ensures existence of a steady state.[2] The flows are *unbalanced*: the arrival rate for the buyers is higher than that for the sellers. This implies that the (flow) Walrasian price is 1.

We explore the limit as the rate of matching $\lambda \to \infty$, which corresponds to the vanishing search frictions. The analysis is expedited by the fact that, due to the continuous time setup, closed-form solutions are available. Our main finding is twofold. If the initial stocks are aligned with the flows, $B(0) \geq S(0)$, then the

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1 The literature includes Wolinsky (1988), De Fraja and Sakovics (2001), Mortensen and Wright (2002), Satterthwaite and Shneyerov (2007, 2008), Atakan (2007a,b), Shneyerov and Wong (2010a,b), Lauermann (2013), Lauermann et al. (2011), among others.

2 Physical death is obviously one possibility, but there may be more mundane reasons for the exit from the market. For a discussion, see Satterthwaite and Shneyerov (2008).

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limit price is Walrasian at all times.[3] If, on the other hand, the initial stocks are misaligned, $B(0) < S(0)$, then there is a *transition period* $[0, t_*)$ along which the limit price is not Walrasian. Thus the alignment of the initial stocks with the flows is both necessary and sufficient for the limit price to be Walrasian at all times.

The intuition for our result is as follows. In our model, rapid matching for the shorter side implies that the stocks of buyers and sellers will be *highly* unbalanced as $\lambda \to \infty$. Since the buyers arrive at a higher rate, after the transition period (if there is any) the buyers will be more abundant, and in fact $B(t)/S(t) \to \infty$ as $\lambda \to \infty$. This means that it will be very difficult for the buyers to find the sellers, and the limit price will be equal to 1. But if the initial stocks are misaligned with the flows, they will have a lasting effect over the transition period. Over this period, the sellers are more abundant, with $B(t)/S(t) \to 0$ as $\lambda \to \infty$, which implies that the sellers will have hard time finding the buyers. In any meeting, the sellers would insist on a price that would give them the discounted value of the surplus they could realize by waiting until t_*. This implies that the limit price over the transition period will be the present discounted value of 1 received at the end of the transition period.

2. Model and equilibrium

Sellers and buyers arrive to the market continuously over time, at rates s and $b > s$ respectively. They meet each other at the rate $\lambda \cdot M(B(t), S(t))$, where $B(t), S(t)$ are the stocks of buyers and sellers in the market at time t and $M(\cdot, \cdot)$ is a matching function.

Assumption 1. The matching function M is continuous on $\mathbb{R}^2_+$, nondecreasing in each argument, and exhibits constant returns to scale (i.e. homogeneous of degree 1), and satisfies $M(B, 0) = M(0, S) = 0$.

It is convenient to define the *rates* at which buyers and sellers meet their partners for the benchmark case $\lambda = 1$,

$$l_B(\zeta) \equiv \frac{M(B,S)}{B} = M(1, \zeta^{-1}), \qquad l_S(\zeta) \equiv \frac{M(B,S)}{B} = M(\zeta, 1),$$

where $\zeta \equiv B/S$ denotes the market tightness.

Remark 1. Assumption 1 implies that $l_S(\cdot)$ is a nondecreasing function, while $l_B(\cdot)$ is a nonincreasing function.

In each meeting, a buyer makes an offer with probability $\alpha_B \in (0, 1)$, and the seller makes an offer with probability $\alpha_S = 1 - \alpha_B$. The offers are made on a take-it-or-leave-it basis. If an offer is accepted, both the traders leave the market forever. If an offer is rejected, both the traders return to the market stock of the traders and resume their participation in the matching process. In order to ensure the existence of a steady state, we assume that the traders leave the market for exogenous reasons at the rate $\delta \geq 0$. Both buyers and sellers discount future at the rate $r > 0$.[4]

Remark 2. If $\delta > 0$, the traders have a finite lifetime in the market, and the stocks of buyers and sellers are bounded at all times, and there will be a steady state. We are also allowing $\delta = 0$. In that case, the market excess of buyers over sellers grows with time without bound.

Each seller has a single, indivisible unit of the good, and values it at 0. Each buyer has a single unit demand and values the good at 1. Since $b > s$, the flow Walrasian price is 1.

A *market equilibrium* is informally defined as a pair of trader utilities $W_B(t)$, $W_S(t)$, or *continuation values*, and the pair of stocks $B(t)$, $S(t)$ that are governed by the trading process. In a subgame-perfect equilibrium of the bargaining game, buyers offer the price that is marginally acceptable to the sellers, i.e. equal to W_S. In their turn, sellers offer the price equal to $1 - W_B$.

$$p_B = W_S, \qquad p_S = 1 - W_B. \tag{1}$$

The offers are accepted.

Therefore, the traders' continuation values satisfy the standard Bellman equations[5]

$$(r+\delta)W_S = \alpha_S \lambda l_S(\zeta)(1 - W_B - W_S) + \dot{W}_S, \tag{2}$$

$$(r+\delta)W_B = \alpha_B \lambda l_B(\zeta)(1 - W_B - W_S) + \dot{W}_B. \tag{3}$$

These equations have the usual "asset pricing" interpretations. For example, the l.h.s. of (2) is equal to the instantaneous return from selling the search "option" and depositing the proceeds at a bank. The r.h.s. describes the instantaneous return from holding the option, which is formed through either trading (the first term), or appreciation over time (the second term). In equilibrium, the l.h.s. and the r.h.s. must be equal.

There are *no* initial conditions for Bellman equations (2) and (3). Their unique solutions will be pinned down by the requirement that the solutions are *bounded*; more precisely,

$$W_B(t) \quad \text{and} \quad W_S(t) \in [0, 1]. \tag{4}$$

Since each meeting results in trade, the rate of increase in the trader stocks are equal to the arrival rate, minus the meeting rate, minus the exogenous exit rate. Stocks of the traders are governed by:

$$\dot{S} = s - \lambda M(B,S) - \delta S, \tag{5}$$

$$\dot{B} = b - \lambda M(B,S) - \delta B, \tag{6}$$

with initial conditions $S(0)$, $B(0)$.

We now show that, for any initial conditions $B(0), S(0) > 0$, there is a unique market equilibrium satisfying Eqs. (2)–(6). The analysis is expedited by the fact that the system that governs the stocks, (5) and (6), is independent of the system for the utilities, (2) and (3).

The following proposition explicitly derives the solution when market tightness function $\zeta(t)$ is exogenously given.

Proposition 1. *For any given market tightness function $\zeta(t)$, a unique bounded solution to the system (2) and (3) is given by*

$$W_S(t) = \int_t^\infty \alpha_S \lambda l_S(\zeta(x))(1 - W(x)) e^{-(r+\delta)(x-t)} dx, \tag{7}$$

$$W_B(t) = \int_t^\infty \alpha_B \lambda l_B(\zeta(x))(1 - W(x)) e^{-(r+\delta)(x-t)} dx, \tag{8}$$

where $W(t) = W_B(t) + W_S(t)$ is the social surplus, given by

$$W(t) = -\int_t^\infty \frac{\lambda\theta(\zeta(x))}{r+\delta+\lambda\theta(\zeta(x))} de^{h(t)-h(x)}, \tag{9}$$

where $\theta(\zeta) \equiv \alpha_B l_B(\zeta) + \alpha_S l_S(\zeta)$ and $h(t) \equiv \int_0^t (r+\delta+\lambda\theta(\zeta(x)))dx$.

[3] The stocks of buyers and sellers at time t are denoted respectively as $B(t)$ and $S(t)$.

[4] This discount and exit rates are assumed to be the same for the buyers and the sellers. The analysis can be easily extended to different discount rates.

[5] Here and below, $\cdot$ denotes a time derivative.