Economics Letters 124 (2014) 318–322

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Persistence under temporal aggregation and differencing

Uwe Hassler *

Goethe University Frankfurt, RuW Building, Grueneburgplatz 1, 60323 Frankfurt, Germany

HIGHLIGHTS

- Cumulating
- Skip sampling
- Difference-stationarity
- Seasonal differencing
- Temporal aggregation

ARTICLE INFO

Article history:
Received 10 February 2014
Received in revised form
7 May 2014
Accepted 11 June 2014
Available online 17 June 2014

JEL classification:
C22
C82

Keywords:
Cumulating
Skip sampling
Difference-stationarity
Seasonal differences

ABSTRACT

Temporal aggregation is known to affect the persistence of time series. We study the aggregation of flow variables as well as stock data, and difference-stationarity is allowed for. Moreover, moving averages encountered when computing annual growth rates (seasonal differences) are investigated. Using a relative persistence measure (long-run variance ratio), it is clarified when persistence is increased or decreased, and by how much. Our results are exact for a finite aggregation level. They are illustrated with monthly time series. Approximate results for the growing aggregation level are provided, too.

1. Introduction

Temporal aggregation is a standard routine when working with time series. Even if the researcher does not aggregate him- or herself, the statistical offices making available the data may only provide aggregates. It has been empirically documented that temporal aggregation affects persistence measures. With the growing aggregation level there are analytical results by Working (1960) and Tiao (1972). For finite aggregation levels, however, there is no clear answer whether persistence is increased or decreased, and by how much. In this letter we give an answer to these questions for the long-run variance ratio as a relative persistence measure. The results are exact for a finite aggregation level and can be approximated with the growing level. We also allow for nonstationary series where differencing is required to obtain stationarity. Moreover, we study the situation where seasonal growth rates are stationary, but annual growth rates (seasonal differences) are computed for convenience. Hassler and Demetrescu (2005) demonstrated experimentally and empirically that annual growth rates will dramatically exaggerate the degree of persistence relative to the persistence present in the seasonal rates. Their arguments are reinforced here theoretically in a general framework.

We study the aggregation of both, stock variables and flow variables. Typical flow data are monthly consumption, where temporal aggregation to quarterly or annual data consists of cumulating the monthly flows to the total quarterly or annual flow. Typical stock series are daily prices or exchange rates. In order to obtain weekly data, one may compute the average of all days of the week, or alternatively, one may take the last weekday as representative of the whole week (called skip sampling).

It has been claimed that the effect of temporal aggregation on persistence in finite samples is an empirical matter; see for instance Rossana and Seater (1995), and more recently Paya et al. (2007), who employed several widely used persistence measures as the sum of autoregressive coefficients, the largest autoregressive

* Tel.: +49 69 798 34762.
E-mail address: hassler@wiwi.uni-frankfurt.de.

http://dx.doi.org/10.1016/j.econlet.2014.06.011
0165-1765/

root and (cumulated) impulse responses: "statistical theory is not definite because some of the results are asymptotic and leave open the question what will happen with actual data", Rossana and Seater (1995, p. 443). Here, we argue that exact theoretical results on the aggregational effect with respect to persistence are readily available when using a *relative* measure, since aggregation affects both, the serial correlation and the variance. We study the long-run variance ratio, which technically equals the normalized spectrum at frequency zero and has been applied e.g. by Cogley and Sargent (2005); see also Cochrane (1988). We find that persistence is not an invariant property of a process, but necessarily linked to the frequency of sampling. In particular, we learn for aggregation of (integrated) stock variables that skip sampling affects persistence very differently from averaging.

The next section provides the notation and assumptions, and reviews alternative aggregation schemes. Section 3 contains the theoretical results on aggregation and differencing with numerical illustrations and discussion. In Section 4, we use monthly exchange rate data to illustrate that our theoretical results may explain well empirical evidence. The concluding section discusses further implications for applied work. Mathematical proofs of Propositions 1 through 3 are relegated to the working paper version of this letter; see Hassler (2013).

2. Notation and assumptions

The univariate time series data are assumed to be generated by a (covariance) stationary process $\{y_t\}$, where the autocovariances at lag h are denoted as

$$\gamma(h) = \mathrm{E}[(y_t - \mathrm{E}(y_t))(y_{t+h} - \mathrm{E}(y_{t+h}))] = \gamma(-h). \tag{1}$$

The expectation $\mu_t = \mathrm{E}(y_t)$ may be constant or not. The stochastic deviations are assumed to follow a regular linear process with an absolutely summable sequence of impulse responses $\{c_j\}$.

Assumption 1. The process $\{y_t\}$, $t \in \mathbb{Z}$, is given by

$$y_t = \mu_t + \sum_{j=0}^{\infty} c_j \varepsilon_{t-j} \quad \text{with} \quad \sum_{j=0}^{\infty} |c_j| < \infty,$$

$$c_0 = 1, \quad \text{and} \quad \sum_{j=0}^{\infty} c_j \neq 0,$$

where $\{\varepsilon_t\}$ is a zero mean white noise process with variance $\mathrm{Var}(\varepsilon_t) = \sigma^2$.

Next to the variance, $\mathrm{Var}(y_t) = \gamma(0)$, we define the long-run variance $\omega^2 = \mathrm{LrV}(y_t)$ depending on all temporal correlations: $\mathrm{LrV}(y_t) = \sum_{h=-\infty}^{\infty} \gamma(h)$. Under the above assumptions it holds that

$$\gamma(0) = \sigma^2 \sum_{j=0}^{\infty} c_j^2 \quad \text{and} \quad \mathrm{LrV}(y_t) = \sigma^2 \left(\sum_{j=0}^{\infty} c_j \right)^2, \tag{2}$$

such that $\mathrm{LrV}(y_t)$ is positive and finite. Aggregation under the limiting cases of $\mathrm{LrV}(y_t) = 0$ or $\mathrm{LrV}(y_t) = \infty$ arising from fractional integration is covered in Souza (2005), Tsai and Chan (2005), and Hassler (2011). Campbell and Mankiw (1987) popularized the cumulated impulse responses as measure of persistence, $\mathrm{CIR}(y) = \sum_{j=0}^{\infty} c_j$. It does not rely on an autoregressive representation of finite order, and has further been advocated by Andrews and Chen (1994) as being superior to the largest autoregressive root in some cases. Note, however, that temporal aggregation will affect both, the variance and the autocovariances, such that a priori the aggregational effect on $\mathrm{CIR}(y)$ is unclear.

In this letter, persistence is measured through the long-run variance ratio:

$$\mathrm{VR}(y) := \frac{\mathrm{LrV}(y_t)}{\mathrm{Var}(y_t)}. \tag{3}$$

This long-run variance ratio $\mathrm{VR}(y)$ has been used by Cochrane (1988), and it equals the normalized spectrum at frequency zero employed by Cogley and Sargent (2005) up to the multiplicative constant $1/2\pi$. In the case of an AR(1) process, $y_t = a\, y_{t-1} + \varepsilon_t$, it holds that

$$\mathrm{VR}(y) = \frac{1-a^2}{(1-a)^2} = \frac{1+a}{1-a} \begin{cases} > 1 & \text{if } a > 0 \\ = 1 & \text{if } a = 0 \\ < 1 & \text{if } a < 0. \end{cases} \tag{4}$$

Hence, we call a process *persistent* if $\mathrm{VR}(y) > 1$, and say it displays *negative persistence* if $\mathrm{VR}(y) < 1$. For applied work, $\mathrm{VR}(y)$ has to be estimated from a sample of size T. The consistent estimation of a (long-run) variance from stationary data is a standard problem of course. Consistent long-run variance estimation is discussed e.g. in Hamilton (1994, Section 10.5).

Clearly, many time series are not stationary. It is often assumed that the observed variable $\{z_t\}$ has to be differenced to obtain stationarity. With the usual difference operator Δ we define

$$\Delta^r z_t = y_t, \quad t = 1, 2, \ldots, T, \tag{5}$$

for some natural number r, where $\{y_t\}$ is a (weakly) stationary sequence characterized in Assumption 1. In the case that $r = 0$, the observable $\{z_t\}$ itself is covariance stationary, while $r = 1$ gives $z_t - z_{t-1} = y_t$. In most applications the order of differencing is 1. In the case of nonstationarity ($r > 0$), the long-run variance ratio is computed in terms of stationary differences: $\mathrm{VR}(\Delta^r z) = \mathrm{LrV}(\Delta^r z_t)/\mathrm{Var}(\Delta^r z_t)$.

Let $\{z_t\}$, $t = 1, 2, \ldots, T$, denote a sample of univariate time series observations to be aggregated over m periods. We assume for simplicity that T is a multiple of m, $T = m\,N$. The aggregate is constructed for the new time scale τ. In the case of flow variables aggregation means cumulating m neighboring, non-overlapping observations to determine the total flow over m sub-periods,

$$\widetilde{z}_\tau := z_{m\tau} + z_{m\tau-1} + \cdots + z_{m(\tau-1)+1}, \quad \tau = 1, 2, \ldots, N. \tag{6}$$

With stock data two aggregation schemes are encountered in practice. Often, stock variables are averaged, which is formally related to the cumulation of stocks with obvious notation: $\overline{z}_\tau = \widetilde{z}_\tau / m$. The usage of the new time scale τ indicates that the averages are not overlapping. Alternatively, stock variables are sometimes aggregated by systematic sampling or skip sampling where only every mth data point is observed,

$$\dot{z}_\tau := z_{m\tau}, \quad \tau = 1, 2, \ldots, N. \tag{7}$$

If the basic variable $\{z_t\}$ is nonstationary as in (5), then the aggregates will be nonstationary, too. Let ∇ stand for the differences operating on the aggregate scale:

$$\nabla \widetilde{z}_\tau = \widetilde{z}_\tau - \widetilde{z}_{\tau-1} \quad \text{and} \quad \nabla \dot{z}_\tau = \dot{z}_\tau - \dot{z}_{\tau-1}.$$

For the differenced aggregates $\{\nabla^r \widetilde{z}_\tau\}$ and $\{\nabla^r \dot{z}_\tau\}$ we define the persistence measures as $\mathrm{VR}(\nabla^r \widetilde{z})$ and $\mathrm{VR}(\nabla^r \dot{z})$. Again, $r = 0$ refers to the situation where $z_t = y_t$ and hence $\widetilde{z}_\tau$ and $\dot{z}_\tau$ are stationary. Generally, it will make a difference whether one first aggregates and differences the aggregates, or the other way round, and the difference shall be spelled out in the next section.

Contrasting the case of non-overlapping averages, $\overline{z}_\tau$, we also consider moving averages of the following type, where L denotes the usual lag operator:

$$S_m(L)\, z_t = z_t + z_{t-1} + \cdots + z_{t-m+1}.$$