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Group size paradox and public goods

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HIGHLIGHTS

- We model an economy with one public good and many private goods.
- We examine the effect of changes in population on total provision of the public good.
- As population grows, the level of provision rises initially and then starts falling.
- This implies inverted-U relationship between group size and public good.
- Technological improvement leads to a reduction in the aggregate provision of the public good.

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ABSTRACT

In an economy with voluntarily provided public goods and private product varieties, and a general class of *CES* preferences, it is shown that aggregate public good contribution follows an inverted-U pattern with respect to group size when private and public goods are substitutable in preferences. With complementarity, however, aggregate provision grows monotonically with group size.

1. Introduction

How group size affects voluntary provision of public goods is an old issue. Olson (1965) had argued that the free-rider problem would worsen in large groups. Chamberlin (1974), McGuire (1974), Bergstrom et al. (1986) and Andreoni (1988), etc. partly countered this view by showing that as the number of agents grew large total public good contribution would approach a finite upper bound. Pecorino (2009a) added to this debate by arguing that more population meant less public goods because individuals switch to greater varieties of private consumption goods that become available with the increased market size.[1]

In this paper, consumers buy a composite private good made up of different varieties of private goods and contribute voluntarily to a public good. In such public good economy, an increase in group size (or, population) will endogenously support a larger variety of private goods, lowering the shadow price of the composite private good. As composite good becomes cheaper, the demand for public good, and consequently its aggregate provision level, will depend on the elasticity of substitution between the composite good and the public good. There is also a traditional income effect resulting from increased group size: the consumers' budgets will be relaxed as any public good level produced in the economy will serve a larger population. So how the combined effects of a larger group size impact on the level of public good will depend very much on the elasticity of substitution between the composite good and the public good, as well as the elasticity of substitution between the various private good varieties.

We show that, when the elasticity of substitution between the composite good and the public good is less than or equal to unity, the conventional wisdom on group-size effect (i.e., larger groups

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[1] Pecorino (2009b) analyzes the effect of group size on public good in a much simpler economy without production but allowing for rivalry in public good's consumption.

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lead to higher public goods) prevails. However, if this elasticity exceeds unity, then often as the group size increases initially the public good level will increase and then decrease. Thus, the relationship between public good and group size exhibits an inverted U-shape.

The model is specified in Section 2, equilibrium analysis appears in Section 3, comparative statics in Section 4, and concluding summary in Section 5.

2. The model

There are L individuals who each inelastically supplies one unit of labor, earns a competitive wage w and spends it on the composite good and contribution toward the public good. Denoting g_j to be the dollar contribution toward the public good by consumer j, $G = \sum_{j=1}^{L} g_j$ is the total voluntary contribution by L consumers. We normalize the price of the public good at unity so that G is the total amount of public good consumed.

The consumers have identical preferences. Representative consumer j solves[2]:

$$\max_{X_j, g_j} U_j = \left[\eta X_j^r + (1-\eta)G^r\right]^{\frac{1}{r}}, \quad 0 \neq r \leq 1,\ 0 < \eta < 1 \tag{1}$$

$$\text{subject to } p_X X_j + g_j = w, \tag{2}$$

$$\text{where } X_j = \Big(\sum_{i=1}^{n} c_{ij}^{\theta}\Big)^{\frac{1}{\theta}}, \quad 0 < \theta < 1, \tag{3}$$

$$\text{and } p_X X_j = \sum_{i=1}^{n} p_i c_{ij}. \tag{4}$$

The composite good X_j for consumer j as defined in (3) is also a *CES* function, of j's consumption of n private goods (c_{ij}) ($i = 1, 2, \ldots, n$). The price of the composite good is denoted by p_X. The price of the private variety i is given by p_i. Finally, we define $\epsilon = \frac{1}{1-r} \geq 0$ as the elasticity of substitution between the composite good and the public good. We also define $\sigma = \frac{1}{1-\theta} > 1$ as the elasticity of substitution between any two private varieties. For the rest of our analysis, we impose the following assumption.

Assumption 1. Suppose $\sigma \geq \epsilon$. That is, within the group the private goods are more substitutable than they are as a group vis-à-vis the public good.

Treating the differentiated private products and the public good to be inherently different (such as different food items vs. community policing), it is natural to assume that the private goods (say, different varieties of food) are more substitutable than they would be as a whole vis-à-vis the public good (i.e., the community policing).

One can have the following solutions from the consumer's problem[3]:

$$G = \frac{w}{\frac{1}{L} + \left(\frac{1-\eta}{\eta} p_X\right)^{-\epsilon} p_X}, \tag{5}$$

$$X_j = \frac{w\left(\frac{1-\eta}{\eta} p_X\right)^{-\epsilon}}{\frac{1}{L} + \left(\frac{1-\eta}{\eta} p_X\right)^{-\epsilon} p_X}, \tag{6}$$

$$c_{ij} = \frac{p_i^{-\sigma}\ (p_X X_j)}{\sum_{k=1}^{n} p_k^{1-\sigma}}, \tag{7}$$

$$p_X = \left(\sum_{i=1}^{n} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{8}$$

This completes the description of the demand side of the model. We now turn to the production side.

3. Monopolistic competition: general equilibrium

There are n varieties of private goods, each produced under monopolistic competition as in Krugman (1980) and Pecorino (2009a). The production technology of good i is:

$$l_i = \alpha + \beta y_i, \quad i = 1, 2, \ldots, n. \tag{9}$$

Here α represents a fixed cost and β is the marginal cost. Total labor employment in the production of n private goods is

$$L_Y = \sum_{i=1}^{n} (\alpha + \beta y_i).$$

For the public good production, we assume a one-to-one transformation from labor to public goods. This along with perfectly competitive production structure implies that the price of the public good becomes its marginal cost of production which is the wage rate (denoted by w). By normalizing the price of the public good, we get $w = 1$. The public good is financed entirely from voluntary contributions. So the total labor employed in public good production is

$$L_G = G.$$

Labor market clearing requires

$$L = L_Y + L_G. \tag{10}$$

With L consumers in the economy and individual demand for the ith variety given by (7), the aggregate demand for the ith variety is $\sum_{j=1}^{L} c_{ij} = c_i L$, suppressing j from c_{ij}. Firm i maximizes profit:

$$\pi_i = p_i y_i - \alpha w - \beta y_i w, \tag{11}$$

$$\text{where } y_i = c_i L. \tag{12}$$

We assume, similar to Krugman (1980) and Pecorino (2009a), that the monopolist treats the price-index (8) as given while maximizing profit.[4] Then the profit-maximizing price can be solved as

$$p_i = \frac{\beta}{\theta}, \quad i = 1, 2, \ldots, n. \tag{13}$$

Free entry and exit in monopolistically competitive industries guarantee zero profit. So setting $\pi_i = 0$ in (11) and noting that $w = 1$, we get

$$y_i = \frac{\theta\alpha}{(1-\theta)\beta}, \quad i = 1, 2, \ldots, n. \tag{14}$$

Using the solved prices and output, obtain $L_Y = \frac{n\alpha}{1-\theta}$. Using (10), the number of differentiated varieties is solved as follows:

$$n = \frac{(L-G)(1-\theta)}{\alpha}. \tag{15}$$

We can also solve for the composite good price-index in (8), using (13), as follows:

$$p_X = n^{\frac{1}{1-\sigma}} \frac{\beta}{\theta}. \tag{16}$$

[2] When $r \downarrow 0$, the utility function (1) approaches the standard Cobb–Douglas form $U_j = X_j^{\eta} G^{1-\eta}$, with unitary elasticity of substitution between the composite good and the public good.

[3] Derivations are available in an online appendix at http://web.iitd.ac.in/~debasis/appendix_groupsizeparadox.pdf (see Appendix A).

[4] Note that the price-index p_X involves the prices p_i's.