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The monetary utility premium and interpersonal comparisons

Jingyuan Li [a], Liqun Liu [b,*]

[a] Department of Finance and Insurance, Lingnan University, Tuen Mun, Hong Kong
[b] Private Enterprise Research Center, Texas A&M University, College Station, TX 77845, USA

HIGHLIGHTS

- The utility premium (UP) is the reduction in expected utility due to an nth-degree risk increase.
- While a very useful concept, the utility premium is not interpersonally comparable.
- The monetary utility premium is UP divided by the expected marginal utility at the initial wealth.
- Comparison of the monetary utility premium is characterized by nth-degree Ross more risk aversion.

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ABSTRACT

The utility premium is generally defined as the pain or reduction in expected utility caused by an nth-degree risk increase, where $n \geq 2$. While it is a very useful concept in understanding a decision maker's choice in uncertain situations, the utility premium is not interpersonally comparable. This note shows that the monetary utility premium – the utility premium divided by the expected marginal utility at the random starting wealth – is interpersonally comparable, and the comparison is characterized by Ross more risk aversion of the corresponding degree.

1. Introduction

The utility premium – the reduction in expected utility caused by an introduction of risk – due to Friedman and Savage (1948) has been revived recently to shed light on decision making in risky environment. For example, Menegatti (2007) and Eeckhoudt and Schlesinger (2009) provide an explanation for the precautionary saving motive using the utility premium. They show that the well-known necessary and sufficient condition for precautionary saving, i.e. the utility function having a positive 3rd derivative, is the same condition ensuring that the utility premium decreases with wealth. When the utility premium decreases with wealth, the individual can reduce the pain – the utility premium – caused by a future labor income risk by saving more now to have more non-labor income in the future.

More generally, the utility premium is also used to refer to the pain or reduction in expected utility caused by an nth-degree risk increase, where $n \geq 2$. Eeckhoudt et al. (2009) and Denuit and Rey (2010) show that the $(n+1)$th-degree risk aversion, i.e. a positive $(n+1)$th derivative if $n+1$ is odd and a negative $(n+1)$th derivative if $n+1$ is even, can be interpreted as the utility premium for an nth-degree risk increase decreasing in wealth.

However, a long-recognized inadequacy of the utility premium is that it depends on the unit with which the utility level is measured. In other words, the utility premium for a given risk introduction or risk increase is not unique under positive linear transformations of the utility function. As a result, the utility premium is not interpersonally comparable.

In contrast, the risk premium – defined as the reduction in the initial wealth a decision maker is willing to accept to avoid a risk introduction or a (nth-degree) risk increase – popularized by Arrow (1971) and Pratt (1964) is invariable to alternative representations of the risk preferences, and hence interpersonally comparable. Indeed, Arrow and Pratt show that $u(x)$ is always willing to pay a weakly larger risk premium to avoid the introduction of a risk than $v(x)$ if and only if $-u''(x)/u'(x) \geq -v''(x)/v'(x)$ for all x. Allowing for random starting wealth levels, Ross (1981) shows that $u(x)$ is always willing to pay a weakly larger risk premium to avoid

* Corresponding author.
E-mail addresses: jingyuanli@ln.edu.hk (J. Li), lliu@tamu.edu (L. Liu).

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a Rothschild and Stiglitz (1970) risk increase than $v(x)$ if and only if $-u''(x)/u'(y) \geq -v''(x)/v'(y)$ for all x and y. Interpersonal risk premium comparisons regarding Ekern's (1980) nth-degree risk increases turn out to be corresponding to the concept of nth-degree Ross more risk aversion: $u(x)$ is always willing to pay a weakly larger risk premium to avoid an nth-degree risk increase than $v(x)$ if and only if $u(x)$ is nth-degree Ross more risk averse than $v(x)$, i.e., $(-1)^{n+1}u^{(n)}(x)/u'(y) \geq (-1)^{n+1}v^{(n)}(x)/v'(y)$ for all x and y, where $u^{(n)}(x)$ and $v^{(n)}(x)$ are respectively the nth-order derivative of $u(x)$ and $v(x)$.[1]

To facilitate interpersonal comparisons of the utility premium, Crainich and Eeckhoudt (2008) propose to use the monetary utility premium – the utility premium divided by the marginal utility – as a measuring stick for risk aversion or downside risk aversion. A recent paper by Huang (unpublished) builds on the work of Crainich and Eeckhoudt and finds that the monetary utility premium for a risk introduction is always weakly larger for $u(x)$ than for $v(x)$ if and only if $-u''(x)/u'(x) \geq -v''(x)/v'(x)$ for all x.[2]

In this note we extend the existing comparative utility premium analysis to utility premiums for Rothschild–Stiglitz's risk increases and, more generally, to utility premiums for Ekern's nth-degree risk increases, where $n \geq 2$. Specifically, for $n \geq 2$, we define the monetary utility premium for an nth-degree risk increase from $\tilde{x}$ to $\tilde{y}$ as $\text{MUP}_u(\tilde{x}, \tilde{y}) = \left[Eu(\tilde{x}) - Eu(\tilde{y})\right]/Eu'(\tilde{x})$. We show that $\text{MUP}_u(\tilde{x}, \tilde{y}) \geq \text{MUP}_v(\tilde{x}, \tilde{y})$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{y}$ is an nth-degree risk increase from $\tilde{x}$ if and only if $u(x)$ is nth-degree Ross more risk averse than $v(x)$, i.e., $(-1)^{n+1}u^{(n)}(x)/u'(y) \geq (-1)^{n+1}v^{(n)}(x)/v'(y)$ for all x and y. A special case of this general result is that when $n = 2$, $\left[Eu(\tilde{x}) - Eu(\tilde{y})\right]/Eu'(\tilde{x}) \geq \left[Ev(\tilde{x}) - Ev(\tilde{y})\right]/Ev'(\tilde{x})$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{y}$ is a R–S risk increase from $\tilde{x}$ if and only if $u(x)$ is Ross more risk averse than $v(x)$, i.e., $-u''(x)/u'(y) \geq -v''(x)/v'(y)$ for all x and y.

In addition to normalizing the utility premium by the marginal utility, as in the monetary utility premium, one can more generally normalize the utility premium by the mth-order derivative of the utility function. We show that similar results hold for the interpersonal comparison of these normalized utility premiums.

2. The monetary utility premium for *n*th-degree risk increases

We begin with the definitions of nth-degree risk aversion and nth-degree risk increases. Let $F(x)$ and $G(x)$ represent the cumulative distribution functions (CDF) of two random variables whose supports are contained in a finite interval denoted $[a, b]$ with no probability mass at point a. This implies that $F(a) = G(a) = 0$ and $F(b) = G(b) = 1$. Letting $F^{[1]}(x)$ denote $F(x)$, higher order cumulative functions are defined by $F^{[k]}(x) = \int_a^x F^{[k-1]}(y)dy,\ k = 2, 3, \ldots$. Similar notation applies to $G(x)$ and other CDFs. Expected utility maximization is assumed. For any utility function $u(x)$: $[a, b] \rightarrow R^1$, assume that $u \in C^\infty$. Denote by $u^{(k)}(x)$ the kth derivative of $u(x)$, $k = 1, 2, 3 \ldots$.

For any integer $n \geq 2$, Ekern (1980) gives the following definition.

Definition 1. (i) Decision maker $u(x)$ is nth-degree risk averse on $[a, b]$ if

$$(-1)^{n+1}u^{(n)}(x) > 0 \quad \text{for all } x \text{ in } [a, b]. \tag{1}$$

(ii) $G(x)$ has more nth-degree risk than $F(x)$ if

$$G^{[k]}(b) = F^{[k]}(b) \quad \text{for } k = 2, \ldots, n, \text{ and} \tag{2}$$

$$G^{[n]}(x) \geq F^{[n]}(x) \quad \text{for all } x \text{ in } [a, b] \text{ with ``}>\text{'' holding for some } x \text{ in } (a, b). \tag{3}$$

Note that the nth-degree risk increase is a special case of nth-degree stochastically dominated change where the first $n - 1$ moments are kept unchanged. When $n = 2$, it is the well-known R–S risk increase; when $n = 3$, it is the downside risk increase due to Menezes et al. (1980). Ekern shows that $G(x)$ has more nth-degree risk than $F(x)$ if and only if every nth-degree risk averse decision maker prefers $F(x)$ to $G(x)$.

We can define the monetary utility premium for nth-degree risk increases that generalizes Crainich and Eeckhoudt's (2008) monetary utility premium for risk increases and downside risk increases.

Definition 2. Suppose $n \geq 2$, and $u(x)$ is increasing and nth-degree risk averse. The monetary utility premium (MUP) for an nth-degree risk increase from $\tilde{x}$ to $\tilde{y}$ is defined as

$$\text{MUP}_u(\tilde{x}, \tilde{y}) = \left[Eu(\tilde{x}) - Eu(\tilde{y})\right]/Eu'(\tilde{x}). \tag{4}$$

Obviously, the monetary utility premium for an nth-degree risk increase is always positive and is measured in the unit of wealth.

The monetary utility premium studied in Huang (unpublished) is a special case of Definition 2 where $n = 2$ and $\tilde{x}$ is degenerate (i.e., the initial wealth is nonrandom). Note that the extension from analyzing risk introductions to analyzing risk increases is significant in that it facilitates further extensions to analyzing higher-degree risk increases.

3. Interpersonal comparison of the monetary utility premium

For two utility functions $u(x)$ and $v(x)$, continue to assume that they are each increasing and nth-degree risk averse on $[a, b]$. The following definition of nth-degree Ross more risk aversion, due to Jindapon and Neilson (2007), generalizes the well-known Ross more risk aversion (Ross, 1981) and Ross more downside risk aversion (Modica and Scarsini, 2005).

Definition 3. $u(x)$ is nth-degree Ross more risk averse than $v(x)$ on $[a, b]$ if

$$\frac{(-1)^{n+1}u^{(n)}(x)}{u'(y)} \geq \frac{(-1)^{n+1}v^{(n)}(x)}{v'(y)} \quad \text{for all } x,\ y \in [a, b],$$

or equivalently, if there exists $\lambda > 0$, such that

$$\frac{u^{(n)}(x)}{v^{(n)}(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)} \quad \text{for all } x,\ y \in [a, b]. \tag{5}$$

It has been shown that $u(x)$ is always willing to pay a weakly larger risk premium to avoid an nth-degree risk increase than $v(x)$ if and only if $u(x)$ is nth-degree Ross more risk averse than $v(x)$ (Li, 2009 and Denuit and Eeckhoudt, 2010). The main result of this paper is the following theorem making a connection between the interpersonal comparison of the monetary utility premium for nth-degree risk increases and the notion of nth-degree Ross more risk aversion. The proof is given in Appendix.

Theorem 1. *For two utility functions $u(x)$ and $v(x)$ defined on interval $[a, b]$ that are each increasing and nth-degree risk averse, the following three statements are equivalent:*

(i) *u is nth-degree Ross more risk averse than v on $[a, b]$, i.e., there exists $\lambda > 0$, such that $\frac{u^{(n)}(x)}{v^{(n)}(x)} \geq \lambda \geq \frac{u'(y)}{v'(y)}$ for all $x,\ y \in [a, b]$.*
(ii) *There exists $\lambda > 0$ and $\phi(x)$: $[a, b] \rightarrow R^1$, such that $u = \lambda v + \phi$, where $\phi'(x) \leq 0$ and $(-1)^{n+1}\phi^{(n)}(x) \geq 0$ for all $x \in [a, b]$.*
(iii) *$\text{MUP}_u(\tilde{x}, \tilde{y}) \geq \text{MUP}_v(\tilde{x}, \tilde{y})$ for all $\tilde{x}$ and $\tilde{y}$ such that $\tilde{y}$ has more nth-degree risk than $\tilde{x}$.*

[1] For studies on nth-degree Ross more risk aversion, see Modica and Scarsini (2005), Jindapon and Neilson (2007), Li (2009), Denuit and Eeckhoudt (2010), and Liu and Meyer (2013b).

[2] For some related studies using the utility premium to help understand risk aversion or downside risk aversion, see Machina and Neilson (1987), Jindapon (2010), Menegatti (2011) and Liu and Meyer (2013a).