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Empirical likelihood-based inference for the generalized entropy class of inequality measures

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HIGHLIGHTS

- We propose an empirical likelihood-based method of inference for the generalized entropy class of inequality measures.
- We conduct a Monte Carlo study to assess the size and power of our proposed test.
- Simulations show that our method matches the performance of the delta method, and in some cases outperforms it.
- We apply our method to some Canadian household income data for illustrative purposes.

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ABSTRACT

We propose an empirical likelihood-based method of inference for comparing inequality between two populations. A series of Monte Carlo experiments are used to assess our method's finite sample performance. We illustrate our approach using some Canadian household income data.

1. Introduction

Ever since the work of Atkinson (1970), there has been significant research interest in economic inequality and poverty. Although the measurement of inequality and poverty are important, statistical inference for such measures have gained considerable interest in recent years. The work of Kakwani (1993), Zheng (2001), Biewen (2002) and Davidson and Flachaire (2007) serve to highlight the importance of statistical inference in measuring inequality and poverty rather than just the incidence.

The growing body of literature surrounding the theory of inequality measurement has been accompanied by increasing availability of income data distribution which have armed researchers with the capability to conduct more sophisticated analyses. Statistical inference for inequality measures was largely neglected until the work of Cowell (1989). Recently, Thompson (2010) derived the asymptotic properties of vector measures of inequality (and poverty). He argued that since there is often no "best" measure of inequality or poverty, multiple measures could be used.

Our method of inference relies on empirical likelihood (EL), a powerful nonparametric statistical method pioneered by Owen (1988, 1990). An advantage of empirical likelihood is that no assumptions are needed regarding the underlying distribution of the data. Thompson (2013) used the approach for making inference on poverty measures which utilize relative poverty lines. His main focus was to compare poverty between two subgroups of a population that share a common poverty line. We depart from focusing on poverty measures and turn our attention to inequality measures (more specifically, we limit our focus to the generalized entropy class of inequality measures).

The remainder of this paper is organized as follows. In Section 2, we provide a brief overview of inequality measures. In Section 3, we present our methodology. In Section 4, we examine the finite sample performance of our method using a Monte Carlo

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simulation. In Section 5, we demonstrate the practicality of our method using an empirical application.

2. Inequality measures

In this section, we provide a basic overview of the measurement of inequality. For a more thorough treatment of the literature, see Cowell (2011) or Cowell (2000). Before proceeding, we need to introduce some notation. Following Thompson (2010), we generalize our approach for vector measures of inequality. Let $Y = (Y_1, \ldots, Y_J)'$ be a random vector whose value is determined by a set of attributes (e.g., income, education, etc.) for an individual from a certain population. In the case where we are interested in only one attribute but we want to consider J distinct measures, we will have $Y_j = Y_k$ for all j, k. Let F_j be the distribution function of Y_j.

There are several different scalar measures of inequality that exist in the literature. We focus exclusively on the generalized entropy class of measures which fulfill the most widely accepted axioms including decomposability (see, e.g., Cowell, 2000).[1] For the random vector Y_j, such measures can be written as $I_j = E_j(h_j(y_j, \mu_j, \alpha_j))$ where E_j denotes expectation under distribution F_j, $h_j(y_j, \mu_j, \alpha_j)$ is some real-valued function, μ_j is the mean of F_j, and α_j is an exogenous parameter (and thus its choice is subjective). Formally, we have

$$I_j = \int h_j(y_j, \mu_j, \alpha_j) dF_j(y_j),$$

where $\mu_j = \int y_j dF_j(y_j)$ and

$$h_j(y_j, \mu_j, \alpha_j) = \begin{cases} [(y_j/\mu_j)^{\alpha_j} - 1]/(\alpha_j^2 - \alpha_j) & \alpha_j \neq 0, 1 \\ -\log(y_j/\mu_j) & \alpha_j = 0 \\ y_j \log(y_j/\mu_j)/\mu_j & \alpha_j = 1. \end{cases}$$

Let $\mu = (\mu_1, \ldots, \mu_J)'$ be the vector of means. A vector of inequality measures can be written as $I = (I_1, \ldots, I_J)'$.

3. Empirical likelihood-based inference

The empirical likelihood method was first brought to the forefront by Owen (1988, 1990). It is a nonparametric method of inference and an alternative to the bootstrap. For an extensive overview, see Owen (2001).

The basic framework can be explained as follows. Let $y_1, \ldots, y_n$ be independent observations with common distribution function F_0. For any distribution function F, let $p_i \geq 0$ be the probability associated with y_i, with $\sum_{i=1}^n p_i = 1$. Define $L(F) = \Pi_{i=1}^n p_i$ as the nonparametric likelihood function for F. Maximizing $L(F)$, subject to the constraints on p_i, yields $p_i = n^{-1}$. In other words, the nonparametric likelihood function attains its maximum when equal weight is placed on each observation.

Let $\theta_0 = T(F_0)$ be a J-dimensional parameter vector for some function T. Analogous to the parametric likelihood case, inferences about θ_0 can be made using the empirical likelihood ratio $L(F)/L(\hat{F})$, where $\hat{F}$ is the empirical distribution function.

Next, suppose we have r estimating functions $g(Y;\theta) = (g_1(Y;\theta), \ldots, g_r(Y;\theta))$ such that $E_F(g(Y;\theta)) = 0$. The main purpose of such functions is to identify the parameters of the problem. The profile empirical likelihood ratio function can then be written as

$$\mathcal{R}(\theta) = \max\left\{ \prod_{i=1}^n np_i \middle| p_i \geq 0,\ \sum_{i=1}^n p_i = 1,\ \sum_{i=1}^n p_i g(y_i;\theta) = 0 \right\}.$$

Under mild regularity conditions, it can be shown that $-2\log\mathcal{R}(\theta_0) \xrightarrow{d} \chi^2_{(J)}$.[2] Details on the computation of the profile likelihood ratio function can be found in Owen (2001, Chapter 3.14).[3]

Our main focus in this paper is to compare inequality between two distinct populations.[4] To distinguish between the two populations, let superscripts A and B hereby indicate association with population A and B, respectively. If we let $D_0 = (D_{1,0}, \ldots, D_{J,0}) = (I_{1,0}^B - I_{1,0}^A, \ldots, I_{J,0}^B - I_{J,0}^A)$, we can test the null hypothesis that $D_0 = D$. Usually, applied researchers would be most interested in testing the null hypothesis that $I_0^A = I_0^B$, which is equivalent to testing $D_0 = 0$. To apply the empirical likelihood-based inference method to the generalized entropy class of inequality measures, we need to encode the parameters of our problem into suitable estimating functions.

Given that we are interested in comparing two populations, the profile empirical likelihood ratio function is

$$\mathcal{R}(\theta^A, \theta^B) = \max\left\{ \prod_{i=1}^{n^A} n^A p_i^A \prod_{i=1}^{n^B} n^B p_i^B \middle| p_i^A \geq 0, p_i^B \geq 0, \right.$$
$$\sum_{i=1}^{n^A} p_i^A = 1,\ \sum_{i=1}^{n^B} p_i^B = 1,$$
$$\left. \sum_{i=1}^{n^A} p_i^A g(y_i^A;\theta^A) = 0,\ \sum_{i=1}^{n^B} p_i^B g(y_i^B;\theta^B) = 0 \right\},$$

where $\theta^A = (\mu^A, I^A)$, $\theta^B = (\mu^B, I^A, D)$, and the estimating functions are

$$g(Y^A;\theta^A) = \begin{pmatrix} Y_1^A - \mu_1^A \\ \vdots \\ Y_J^A - \mu_J^A \\ h_1(Y_1^A, \mu_1^A, \alpha_1) - I_1^A \\ \vdots \\ h_J(Y_J^A, \mu_J^A, \alpha_J) - I_J^A \end{pmatrix},$$

and

$$g(Y^B;\theta^B) = \begin{pmatrix} Y_1^B - \mu_1^B \\ \vdots \\ Y_J^B - \mu_J^B \\ h_1(Y_1^B, \mu_1^B, \alpha_1) - I_1^A - D_1 \\ \vdots \\ h_J(Y_J^B, \mu_J^B, \alpha_J) - I_J^A - D_J \end{pmatrix}.$$

Since we are only interested in conducting hypotheses on D, the remaining parameters in the μ^A, μ^B and I^A vectors are regarded as "nuisance" parameters. Following Owen (1990), we can "profile out" such parameters by maximizing over them. So the empirical likelihood ratio function for D is

$$\mathcal{R}(D) = \max_{\mu^A, \mu^B, I^A} \mathcal{R}(\mu^A, \mu^B, I^A, D).$$

To compute $\mathcal{R}(D)$ for any vector D, we can follow Owen (1990) and use a nested algorithm which involves an "inner" and "outer"

[1] The Atkinson class of inequality measures, and the Gini index are some of the other well established measures of inequality.

[2] A bootstrap calibration is also possible (see Owen, 2001, Chapter 3.3).

[3] Computational routines for several statistical packages are available on Owen's website: http://www-stat.stanford.edu/~owen/empirical/.

[4] There have been numerous studies done on empirical likelihood for the two population case (see, e.g., Wu and Yan, 2012).