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An exploration of the effect of doubt during disasters on equity premiums

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HIGHLIGHTS

- We consider the effect on equity premiums of a doubt during disasters.
- We derive analytical solutions of equity premiums in the case of power utility.
- We conduct numerical exercises in the case of the recursive utility.
- The recursive utility model with a high IES generates high equity premium.
- Ignoring doubt during disasters biases computed equity premiums downward.

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ABSTRACT

In this note, we consider the effect on equity premiums of a representative household's subjective expectations during disasters. In particular, we focus on the effect of doubt during disasters. We derive analytical solutions of equity premiums in the model of power utility function and conduct numerical exercises of the model of the recursive utility function. Our contribution is to demonstrate that doubt during disasters – even mild ones – generates high equity premiums.

1. Introduction

Rietz (1988) and Barro (2006) argue that a disaster risk generates large equity premiums even in an exchange economy with a representative agent. Recently, many researchers have explored the effects of various empirical characterizations of disasters on equity premiums.[1] In most of these studies, the rational expectations model is used. However, the problem is that the stochastic processes of disasters might be unknown because of their infrequency. Given that disasters tend to unfold over a number of periods and are followed by recoveries, households may be unable to predict accurately the associated increases in the volatility of consumption growth rates.

This note examines how doubt, which is an example of the type of subjective expectation proposed by Abel (2002), affects equity premiums. Doubt is modeled by using the mean-preserving spread of the objective distribution. We demonstrate that whether doubt during disasters generates high equity premiums depends on the value of the intertemporal elasticity of substitution (IES). In particular, we demonstrate analytically that if the model incorporates a power utility function, doubt during disasters lowers equity premiums. However, if the model incorporates the recursive utility function proposed by Epstein and Zin (1991) and Weil (1989), with a high IES, doubt during even mild disasters generates high equity premiums.

2. The model and its equilibrium

2.1. Modeling doubt during disasters

We model an economy in which a representative agent consumes fruit from Lucas trees. We assume that the number of Lucas

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[1] Gourio (2008) considers the recoveries that follow disasters. Saito and Suzuki (2014) explore the persistent disasters.

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trees is constant. We let C_t and A_t denote consumption and output in period t, respectively. Because we model a closed economy, output equals consumption: $C_t = A_t$.

The two states, $s_t \in \{n, d\}$, are the normal and disaster states, denoted by n and d, respectively. Output depends on what state the economy is in: $A_t = A(s_t)$. The stochastic process for the logarithm of output is:

$$\ln A(s_{t+1}) = \ln A(s_t) + g + u_{t+1} + \ln(1-b)\xi(s_{t+1}) + v_{t+1}\zeta(s_t),$$

where g is the trend growth rate and b denotes the scale of the disaster. If there is a disaster in period $t+1$ (i.e., $s_{t+1} = d$), then $\xi(s_{t+1}) = 1$, but otherwise (i.e., $s_{t+1} = n$), $\xi(s_{t+1}) = 0$. In the normal state, disasters occur with a probability of ϕ. In disaster states, disasters are not repeated. Therefore, the stationary probability of the economy being in a disaster state is $\psi = \frac{\phi}{1+\phi}$. The term u_{t+1} is an independent and identically distributed normal shock with a distribution of $N(0, \sigma_u^2)$. Although disasters are one-off events, uncertainty about output growth rates increases following disasters. The term v_{t+1} represents doubt about output growth rates following a disaster. If a disaster occurs in the current period ($s_t = d$), then $\zeta(s_t) = 1$, but otherwise ($s_t = n$), $\zeta(s_t) = 0$. We make the assumption below about doubt during disasters.

Assumption 1. Following Abel (2002), subjective doubt during disasters is modeled by using the mean-preserving spread. That is, the distribution of v_{t+1} is assumed to be $N(-\frac{\sigma_v^2}{2}, \sigma_v^2)$.

The subjective expectations operator conditional on the current state s is denoted by $E_s^*[\cdot]$. The output growth rate from state s to state s' is $\frac{A(s')}{A(s)}$. The conditional expectation of the growth rate, $\bar{A}_{ss'} \equiv E_s^*\left[\frac{A(s')}{A(s)}\right]$, is $\bar{A}_{nn} \equiv \exp\{g + \frac{\sigma_u^2}{2}\}$ when there are no disasters, and $\bar{A}_{nd} \equiv (1-b)\bar{A}_{nn}$ when there is a disaster. The conditional expected growth rate prevailing after a disaster is denoted as $\bar{A}_{dn} \equiv E_d^*\left[\frac{A_n}{A_d}\right]$. Because doubt is modeled by using the mean-preserving spread, doubt does not affect the expected growth rates of output; $\bar{A}_{dn} = \exp\{g + \frac{\sigma_u^2}{2}\}$.

2.2. Equity premiums with doubt during disasters

As Abel (2002) explains, asset prices and returns are determined by the Euler equation under subjective probabilities. Suppose that m_{t+1} denotes the stochastic discount factor (SDF). Then, the return on asset i, denoted by R_{t+1}^i, must satisfy the following pricing equation:

$$1 = E_t^*[m_{t+1}R_{t+1}^i]. \tag{1}$$

We assume that disasters follow a Markov process and that the representative agent has a power utility function or a recursive utility function. In this case, asset prices are a function of the states.

The return on a safe asset is determined by the following equation:

$$R_s^f = \frac{1}{E_s^*[m_{ss'}]}, \tag{2}$$

where $m_{ss'}$ denotes the SDF as one moves from state s to s'. Although the return on a safe asset depends on the state of the economy, it is known in period t. The unconditional expectation of the return on a safe asset is $R^f = (1-\psi)R_n^f + \psi R_d^f$.

The price of a Lucas tree in state s is P_s^e. By using the price–dividend ratio in each state s, defined as $\omega_s \equiv \frac{P_s}{A_s}$, we can represent the ex post return on equity as one moves from state s to s' as follows:

$$R_{ss'}^e = \frac{A(s')}{A(s)}\frac{\omega_{s'}+1}{\omega_s}. \tag{3}$$

From Eq. (1) and the above definition, we can derive the following equation:

$$\omega_s = E_s^*\left[m_{ss'}\frac{A(s')}{A(s)}(\omega_{s'}+1)\right]. \tag{4}$$

We can use this equation to compute the price–dividend ratio in each state.

The expected rates of returns on equity can be written as follows:

$$R_n^e = (1-\phi)\bar{R}_{nn}^e + \phi\bar{R}_{nd}^e \tag{5}$$

$$= \bar{A}_{nn}\left[(1-\phi)\frac{\omega_n+1}{\omega_n} + \phi(1-b)\frac{\omega_d+1}{\omega_n}\right] \tag{6}$$

$$R_d^e = \bar{R}_{dn}^e \tag{7}$$

$$= \bar{A}_{dn}\frac{w_n+1}{w_d}, \tag{8}$$

where $\bar{R}_{nn}^e \equiv E_n^*[R_{nn}^e]$, $\bar{R}_{nd}^e \equiv E_n^*[R_{nd}^e]$, and $\bar{R}_{dn}^e \equiv E_d^*[R_{dn}^e]$.[2] The unconditional expected equity return is $R^e = (1-\psi)R_n^e + \psi R_d^e$.

Thus, the unconditional expected equity premium is:

$$\pi = R^e - R^f.$$

Simple manipulation of this expression yields the following unconditional expected equity premium:

$$\pi = -(1-\psi)\frac{\mathrm{cov}_n^*[m_{ns'}, R_{ns'}^e]}{E_n^*[m_{ns'}]}, \tag{9}$$

where $\mathrm{cov}_n^*[\cdot]$ denotes the subjective covariance operator conditional on the state n.

2.3. The power utility function

Based on a power utility function, the SDF is $m_{t+1} = e^{-\rho}\left[\frac{A(s_{t+1})}{A(s_t)}\right]^{-\gamma}$, where $-\rho$ denotes the subjective discount rate and γ denotes the coefficient of relative risk aversion (CRRA), which is equivalent to the reciprocal of the IES. Given Eq. (4), the price–dividend ratios in the normal and disaster states, respectively, solve the following equations:

$$\omega_n = \beta[(1-\phi)(\omega_n+1) + \phi\delta(\omega_d+1)]$$
$$\omega_d = \beta\alpha(\omega_n+1),$$

where $\beta \equiv \exp\{-\rho + (1-\gamma)g + (1-\gamma)^2\frac{\sigma_u^2}{2}\}$, $\alpha \equiv \exp\{-\gamma(1-\gamma)\frac{\sigma_v^2}{2}\}$, and $\delta \equiv (1-b)^{1-\gamma}$. The price–dividend ratios in both states can be written as follows:

$$\omega_n = \frac{\beta(1-\phi+\phi\delta) + \beta^2\phi\delta\alpha}{1-\beta(1-\phi)-\beta^2\phi\delta\alpha}$$

$$\omega_d = \frac{(1+\phi\delta\beta)\beta\alpha}{1-\beta(1-\phi)-\beta^2\phi\delta\alpha}.$$

[2] Abel (2002) argues that sample moment of equilibrium rate of return on assets can be computed using the objective distributions if there is a very long time series of observations. However, because disasters are infrequent events, it may be difficult to determine the true objective distribution of the consumption growth rate's volatility immediately after a disaster. Therefore, our analysis focuses on the subjective expected returns on assets.