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Equilibrium bids in practical multi-attribute auctions☆

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HIGHLIGHTS

- We propose a nonlinear scoring function to evaluate all the bids.
- The components of the bids are transformed into comparable ones.
- We characterize the equilibrium bidding strategy.
- The equilibrium quality improves as the number of bidders increases.
- The equilibrium price decreases as the number of bidders increases.

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ABSTRACT

This article proposes a nonlinear scoring rule which transforms multiple attributes of a bid into comparable dimensionless ones. Practically, the buyer can use it to select the most competitive winner. For risk-neutral bidders, we characterize a symmetric Bayes–Nash equilibrium and find that as the number of bidders increases the equilibrium quality improves, whereas the equilibrium price decreases.

1. Introduction

The multi-attribute auction was ever mentioned by McAfee and McMillan (1987) and examined by Thiel (1988) using consumer theory. The seminal paper by Che (1993) among the multi-attribute auction literature systematically analyzed bidding behavior with the standard methodology on price-only auctions. In Che's model, a bid contains two dimensions of price and quality: each bidder's type is unidimensional, and suppliers' cost functions are independent of each other. Branco (1997) extended Che's independent-cost model to the correlated-cost case, David et al. (2006) and Nishinura (2012) generalized Che's work from one non-price attribute (quality) to a number of such attributes, and Asker and Cantillon (2008, 2010) considered a procurement auction where each bidder's type is multidimensional. With unidimensional type parameter, Hanazonoy et al. (2012) extended the approach by Che (1993) and Asker and Cantillon (2008) by taking the quality offered and the supplier's cost as endogenous. In this article, the model described is similar to Che's first-score case: ex ante symmetric, risk neutral bidders have a cost function which is parameterized via a unidimensional type and submit their two-dimensional bids of price and quality.

To select a winner, Che (1993), Branco (1997), David et al. (2006), Asker and Cantillon (2008) and Nishinura (2012) used the quasilinear scoring rule that is linear in price,[1] while Hanazonoy et al. (2012) mapped the multidimensional bid onto the unidimensional score according to the more realistic price–quality ratio scoring rule. Practically, the unit of price is different from that of quality, and sometimes, quality cannot be evaluated by equivalent money. Even if quality can be monetized, when the value of

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[1] The scoring rule in Che (1993) is assumed to be $S(p, q) = s(q) - p$, where p is the price and q is the quality. In Branco (1997), it equals $S(t, q) = V(q) - (1 + \lambda)t$, where t is the bidder's expected payment and q is the quality. In Asker and Cantillon (2008) and Nishinura (2012), it is similar to Che (1993). In David et al. (2006), it is $S(p, q_1, \ldots, q_m) = -p + \sum_{j=1}^{m} \omega_j \sqrt{q_j}$, where p is the price, q_j is the quality and ω_j is the weight assigned to quality j, $j = 1, 2, \ldots, m$.

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price is a thousand times of the value of quality, the quality has little contribution to the score of a bid. Thus, the quasilinear scoring rule weakens bidders' efforts in quality attributes. As long as two bids have the same price, they have almost the same score. At the moment, a multi-attribute auction reduces to a price-only auction since the winner is almost determined by the price. In this article, we propose a new scoring rule that is nonlinear in price. Compared with the quasilinear scoring rule, our scoring rule not only eliminates the problem caused by different units of price and quality, but also transforms different attributes of a bid into comparable dimensionless ones and further emphasizes on bidders' effort to each dimension of the bid.

With the scoring rule being quasilinear, the optimal quality for a winning bidder is independent of the number of bidders (see, e.g., Lemma 1 in Che, 1993, Eq. (13) in Branco, 1997, Lemma 2 in David et al., 2006, and Lemma 1 in Nishinura, 2012). Thus, having more rivals does not drive a bidder to offer higher qualities. In contrast, this article characterizes a symmetric Bayesian Nash equilibrium which drives every type of a bidder to pledge to supply higher quality given higher number of bidders. This result could formalize the intuition that competition may give rise to higher qualities, an intuition that the related work failed to capture due to its particular quasilinear scoring rule.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes a symmetric Bayesian Nash equilibrium with interior-solution bid and examines its properties.

2. The model

Consider the case of an indivisible contract procurement. The buyer faces n bidders indexed by $i = 1, 2, \ldots, n$, specifies bidding attributes consisting of price (p) and quality (q), and announces the scoring function $S(p, q)$ to bidders. Bidder i then submits a sealed-bid (p_i, q_i). To avoid bids of high price and inferior quality, we assume that the buyer only accepts those bids for which p_i must be less than the highest acceptable bidding price $\overline{p}$ and q_i must be higher than the lowest acceptable quality $\underline{q} > 0$. Each (p_i, q_i) is evaluated according to $S(p, q)$. The bidder who scores highest wins and then sells the good according to the price and quality specified in her winning bid.

To eliminate computational problems caused by different units of price and quality and obtain comparable scales, (p_i, q_i) is normalized as (p_i^*, q_i^*), where $p_i^* = \overline{p}/p_i$ and $q_i^* = q_i/\underline{q}$. As a result, each element of (p_i^*, q_i^*) has the dimensionless unit, and $p_i^* \geq 1$, $q_i^* \geq 1$. The score for (p_i^*, q_i^*) is defined as $S_i = \omega_1 p_i^* + \omega_2 q_i^*$, where weights ω_1 and ω_2 satisfy $\omega_1 + \omega_2 = 1$.

Suppose that bidders' production efficiency is determined by the cost parameter θ (type) which is private information. A larger θ represents a higher efficiency. As in Che (1993), we assume that θ is independently and identically distributed over $[\underline{\theta}, \overline{\theta}]$$(0 < \underline{\theta} < \overline{\theta} < +\infty)$, following the distribution function F with density $f > 0$. Denote $G(\cdot) = F^{n-1}(\cdot)$ and $G'(\cdot) = g(\cdot)$. Let bidders' cost function be $c(q; \theta)$. Practically, the higher the production efficiency is, the lower the cost and marginal cost of quality become, i.e., $c_\theta < 0$ and $c_{q\theta} \leq 0$; the higher the quality that is offered, the higher the production cost is and the lower the marginal cost of quality is, i.e., $c_q > 0$ and $c_{qq} < 0$. Assume that $p'(\theta) < 0$ and $q'(\theta) > 0$.

3. Equilibrium bidding strategy

Based on the type θ, the risk neutral bidders choose bids (p, q) to maximize their expected profits. Then, we have the following Theorem 1.

Theorem 1. *For risk-neutral bidders, the symmetric Bayes–Nash equilibrium bidding strategies, $(p(\theta), q(\theta))$, with interior-solution bids are given by*[2]

$$p(\theta) = c(q(\theta); \theta) - \int_{\underline{\theta}}^{\theta} c_\theta(q(x); x)\left(\frac{F(x)}{F(\theta)}\right)^{n-1} dx, \tag{1}$$

$$Ac_q^{\frac{1}{2}}(q(\theta); \theta) + \int_{\underline{\theta}}^{\theta} c_\theta(q(x); x)\left(\frac{F(x)}{F(\theta)}\right)^{n-1} dx - c(q(\theta); \theta) = 0, \tag{2}$$

with the boundary condition of

$$Ac_q^{\frac{1}{2}}(q(\underline{\theta}); \underline{\theta}) - c(q(\underline{\theta}); \underline{\theta}) = 0, \tag{3}$$

where $A = \left(\frac{\omega_1}{\omega_2}\overline{p}\underline{q}\right)^{\frac{1}{2}}$.

Proof. If bidder 1 with θ bids (p, q) and wins, she obtains a score $S_1 = \omega_1\overline{p}/p + \omega_2 q/\underline{q}$ and earns a profit $p - c(q; \theta)$. Her winning probability is $\text{Prob}\{S(\theta_i) < S_1, i = 2, \ldots, n\} = G(S^{-1}(S_1))$. Thus, bidder 1's objective function can be expressed as

$$\pi(p, q; \theta) = \Big[p - c(q; \theta)\Big]G\Big[S^{-1}\Big(\omega_1\frac{\overline{p}}{p} + \omega_2\frac{q}{\underline{q}}\Big)\Big]. \tag{4}$$

Differentiating (4) with respect to p and q, respectively, yields

$$\begin{cases} \dfrac{\partial\pi(p, q; \theta)}{\partial p} = G(S^{-1}(S_1)) \\ \qquad -[p - c(q; \theta)]g(S^{-1}(S_1))\dfrac{1}{S'(S^{-1}(S_1))}\dfrac{\omega_1\overline{p}}{p^2}, \\ \dfrac{\partial\pi(p, q; \theta)}{\partial q} = -c_q(q; \theta)G(S^{-1}(S_1)) \\ \qquad +[p - c(q; \theta)]g(S^{-1}(S_1))\dfrac{\omega_2}{\underline{q}S'(S^{-1}(S_1))}. \end{cases} \tag{5}$$

At a symmetric Bayes–Nash equilibrium with interior-solution bids, bidder 1's optimal choice must be to bid $(p(\theta), q(\theta))$ and $(\partial\pi(p, q; \theta)/\partial p, \partial\pi(p, q; \theta)/\partial q)|_{(p=p(\theta), q=q(\theta))} = (0, 0)$. Since $S^{-1}(S_1) = \theta$, it follows from (5) that bidder 1's bid $(p(\theta), q(\theta))$ must satisfy

$$\begin{cases} G(\theta) = [p(\theta) - c(q(\theta); \theta)]g(\theta)\dfrac{\omega_1\overline{p}}{p^2(\theta)S'(\theta)} \\ c_q(q(\theta); \theta)G(\theta) = [p(\theta) - c(q(\theta); \theta)]g(\theta)\dfrac{\omega_2}{\underline{q}S'(\theta)}. \end{cases} \tag{6}$$

From (6), we have

$$p^2(\theta) = A^2 c_q(q(\theta); \theta), \tag{7}$$

where $A = \left(\frac{\omega_1}{\omega_2}\overline{p}\underline{q}\right)^{\frac{1}{2}}$. Substituting $S'(\theta) = -\omega_1\overline{p}p'(\theta)/p^2(\theta) + \omega_2 q'(\theta)/\underline{q}$ into the second equation of (6) yields

$$\Big[G(\theta)p(\theta)\Big]' = c(q(\theta), \theta)g(\theta) + G(\theta)c_q(q(\theta), \theta)q'(\theta). \tag{8}$$

Integrating both sides of (8) with respect to θ over $[\underline{\theta}, \theta]$, yields

$$G(\theta)p(\theta) = G(\underline{\theta})p(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} c(q(x); x)g(x)dx + \int_{\underline{\theta}}^{\theta} G(x)c_q(q(x); x)q'(x)dx$$

[2] There may be an equilibrium with some corner-solution bids where Eqs. (1) and (2) do not hold.