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Contest design and heterogeneity

Rudi Stracke *

University of Munich (LMU), Schackstr. 4, D-80539 Munich, Germany

HIGHLIGHTS

- This paper compares two prominent contest structures.
- Contestants are heterogeneous in contrast to much of the existing literature.
- Comparison indicates that the adverse effect of heterogeneity is structure specific.
- Model can rationalize experimental evidence on comparison of different contest structures.

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ABSTRACT

I show that the effect of heterogeneity on contest investments depends on the structure of the competition, which implies that heterogeneity matters for optimal contest design. This insight helps to explain empirical evidence on the comparison of different contest structures.

1. Introduction

Gradstein and Konrad (1999) were the first to analyze how the structure of matches affects the behavior of contestants. Ever since, the question of how to optimally structure the competition between a given number of contestants is an integral part of the contest design literature. However, even though probably most contests are among unequal contestants, the literature on imperfectly discriminating contests assumes that the field of contestants is perfectly homogeneous (Gradstein and Konrad, 1999; Fu and Lu, 2012). This paper is a first attempt to close this gap in the literature.

I analyze the behavior of four heterogeneous contestants in a 'grand' and a 'sequential pairwise elimination' Tullock (1980) lottery contest. The comparison of total contest investments in the two formats indicates that structural parameters can be chosen in such a way as to moderate (or strengthen) the detrimental effect of heterogeneity on contest investments. In particular, the results of the comparison suggest that the effect of heterogeneity on investment incentives is weaker in the sequential than in the grand contest.

Different contest structures with heterogeneous contestants have so far only been considered for the perfectly discriminating all-pay auction by Moldovanu and Sela (2006). The authors assume that abilities are private information of contestants, however, such that their results might be driven by a mixture of type uncertainty and type heterogeneity. I assume that types are common knowledge among contestants to allow for a clean identification of structure specific heterogeneity effects.

2. Model and equilibrium

Consider the simplest case which allows for a comparison of the two structures with four risk-neutral contestants (Fig. 1). They interact simultaneously in the grand contest, while there are three pairwise interactions on two separate stages in the sequential format. Any pairwise interaction of the sequential format, as well as the simultaneous interaction between four contestants in the grand contest is modeled as a Tullock (1980) lottery contest with complete information and linear investment costs.

Contestants are of different types: some attach a high value v_H to winning the contest, while winning is worth v_L for low valuation contestants ($v_H > v_L > 0$).[1] Let $n \in \{0; 1; 2; 3; 4\}$ denote the

* Tel.: +49 89 80046713.
E-mail address: Rudi.Stracke@lmu.de.

[1] The choice of the heterogeneity parameter (valuation, productivity, cost) does not affect the results.

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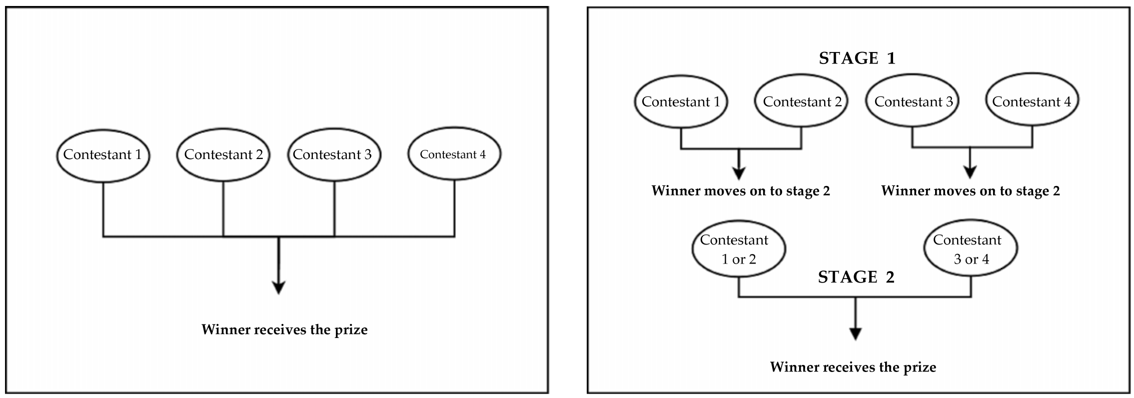

(a) Grand contest.

(b) Sequential elimination contest.

Fig. 1. Design options.

number of low valuation contestants who compete with $4-n$ high valuation opponents. Then, the parameter n characterizes each of the five meaningful configurations of the grand contest. There are six different configurations in the sequential contest, however, since seeding of types matters for $n = 2$. To ensure comparability of both structures, I assume that the seeding of types is random.[2]

2.1. Grand contest

Contestant i maximizes her expected payoff by choosing investment x_i, taking as given investments into the contest by opponents j, k, and l; the solution concept is Nash Equilibrium. The optimization problem of i with valuation v_m ($m = \{\mathtt{H}, \mathtt{L}\}$) reads

$$\max_{x_i \geq 0} \Pi_i[x_i, x_j, x_k, x_l] = \frac{x_i}{X} v_m - x_i, \tag{1}$$

where $X = x_i + x_j + x_k + x_l$. Since (1) is concave in x_i, equilibrium contest investments in this game are characterized by a system of four first-order conditions and four participation constraints. High valuation types will always participate in equilibrium and optimal investments do not differ between contestants of the same type. This leaves a system of two first-order conditions (one for each type) and one participation constraint for low valuation contestants. Equilibrium investments for both types are derived in the appendix.

2.2. Sequential elimination contest

Since the Subgame Perfect Nash Equilibrium in the sequential contest is obtained through backward induction, I start with the stage-2 subgame.

Stage 2. Consider the optimization problem of contestant i who maximizes her expected payoff in a simultaneous two player contest by investing x_{i2}, taken the investment of opponent k as given:

$$\max_{x_{i2} \geq 0} \Pi_{i2}(x_{i2}, x_{k2}) = \frac{x_{i2}}{X} v_m - x_{i2},$$

where i attaches the value v_m ($m = \{\mathtt{H}, \mathtt{L}\}$) to winning the contest, and $X = x_{i2} + x_{k2}$. Equilibrium investments in the homogeneous ($\mathtt{HH}$, $\mathtt{LL}$) and the heterogeneous ($\mathtt{HL}$) interactions are determined by first-order conditions (Nti, 1999) and derived in the appendix.

Stage 1. Assume that contestants i and j, as well as contestants k and l, compete with each other for the right to move on to the next stage in the two parallel stage-1 interactions. Consider the optimization problem of contestant i, which reads

$$\max_{x_{i1} \geq 0} \Pi_{i1}(x_{i1}, x_{j1} | x_{k1}, x_{l1}) = \frac{x_{i1}}{x_{i1} + x_{j1}} \times \underbrace{\left[\frac{x_{k1}}{x_{k1} + x_{l1}} \Pi_{i2}(x_{i2}^*, x_{k2}^*) + \frac{x_{l1}}{x_{k1} + x_{l1}} \Pi_{i2}(x_{i2}^*, x_{l2}^*) \right]}_{C_i(x_{k1}, x_{l1})} - x_{i1}.$$

Contestant i maximizes her expected payoff Π_{i1} by choosing contest investment x_{i1}. The (continuation) value of a participation in stage 2 for contestant i, denoted C_i, depends on the type of the potential stage-2 opponents k and l, since their type determines the expected stage-2 payoffs $\Pi_{i2}(x_{i2}^*, x_{k2}^*)$ and $\Pi_{i2}(x_{i2}^*, x_{l2}^*)$, respectively. It is endogenously determined by investments x_{k1} and x_{l1} of contestants k and l in the parallel interaction whenever k and l are different types. Equilibrium stage-1 investments are characterized by a system of four first-order optimality conditions and derived in the appendix.

3. Results

Comparing the two structures. With homogeneous contestants, total contest investments (TCI) are the same in both structures for the lottery contest technology (Gradstein and Konrad, 1999). Thus, a comparison of TCI in heterogeneous configurations of the grand and the sequential contest allows for a clean identification of the structure specific effect of heterogeneity—everything is held constant but the response of contestants to heterogeneity, which may or may not depend on the structure of the competition. The comparison delivers:

Results 1. *In interior equilibria, total contest investments are strictly higher in the sequential elimination than in the grand contest for* $n \in \{1; 2; 3\}$.

Proof. See the Appendix. □

It is an immediate consequence of Result 1 that the adverse effect of heterogeneity on overall investment incentives is less pronounced

[2] Random seeding implies that the seeding with homogeneous (heterogeneous) stage-1 interactions occurs with probability 1/3 (2/3). This assumption is without loss of generality, since Result 1 holds in both seedings. Höchtl et al. (2011) compare total effort in the two seedings.