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Alternative unit root testing strategies using the Fourier approximation

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HIGHLIGHTS

- We use the Fourier Dickey–Fuller (FDF) statistic for a unit root test.
- We consider three different FDF testing strategies: pretesting, union of rejection, and hybrid.
- We show, by simulation, that the hybrid strategy generally outperforms the other two strategies.

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ABSTRACT

Two alternatives to Enders and Lee's (2012a,b) Fourier unit root testing strategy, which incorporates pretesting for nonlinearity, are considered. One is based on the union of rejection (UR) approach, and the other is a hybrid strategy that combines the UR approach with the use of extra information from nonlinearity pretesting. Simulation results show that the two proposed strategies, especially the hybrid, frequently outperform the original pretesting strategy.

1. Introduction

In a series of recent papers, Enders and Lee (2012a,b) and Rodrigues and Taylor (2012) propose a new approach to the issue of structural breaks in unit root testing: they suggest using a Fourier approximation to the deterministic trend component of the series being tested. In contrast to the conventional approach of Perron (1989) and others, the new Fourier Dickey–Fuller (FDF) tests, as pointed out in Enders and Lee (2012b), adopt a smaller number of parameters to approximate unknown multiple breaks. Thus, the number of breaks is not fixed *a priori*. Moreover, they allow for breaks under both the null hypothesis and the alternative hypothesis. As such, they are free of the spurious rejections problem which can be found in the popular endogenous tests of Zivot and Andrews (1992) and Lumsdaine and Papell (1997); see, for example, Nunes et al. (1997).

Of course, in practice, whether the trend function is linear or nonlinear (with breaks) is typically unknown *a priori*. If the trend is actually linear, a test incorporating an unnecessary Fourier component can entail a power loss. To avoid this unfavourable outcome, Enders and Lee (2012a,b) propose a unit root testing strategy conditioned on "pretesting" for the presence of nonlinearity. The FDF test is recommended if linearity is rejected; otherwise, the original Dickey–Fuller (DF) test is preferred.

The Enders and Lee (EL) pretesting procedure is very useful, but it tends to be over-sized when the series being tested is linear. Further, it involves a non-trivial loss of power in cases where the pretest fails to reject linearity when the trend is indeed nonlinear (see Enders and Lee (2012b, Table 4)). In this paper, we suggest conducting the FDF test via two alternatives to the EL pretesting procedure. First, we consider the union of rejection (UR) strategy proposed by Harvey et al. (2009) for choosing between the DF and FDF tests. Basically, the UR strategy consists of the simple decision rule "reject the unit root null if either the DF test or the FDF

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Table 1
Critical values—union of rejection.

(%)	Trend				Level			
	$T = 100$	$T = 200$	$T = 500$	$T = 2500$	$T = 100$	$T = 200$	$T = 500$	$T = 2500$
1	−4.45	−4.37	−4.33	−4.31	−3.83	−3.81	−3.78	−3.76
5	−3.87	−3.82	−3.80	−3.79	−3.24	−3.23	−3.22	−3.21
10	−3.57	−3.54	−3.52	−3.52	−2.95	−2.94	−2.93	−2.91

test rejects" at a given significance level with size-corrected critical values. Second, we adopt a "hybrid" approach suggested in Harvey et al. (2012) which incorporates the UR strategy with the extra information obtained from nonlinearity pretesting. Specifically, when nonlinearity is evident, the FDF test is recommended; however, the UR strategy is preferred if the pretest cannot reject linearity. We find, via simulation, that the UR and hybrid approaches tend to achieve better size properties than the pretesting procedure (though the UR strategy can, at times, be rather conservative). In terms of power, the hybrid approach frequently dominates the UR strategy and exhibits competitive power compared with the pretesting procedure.

2. Alternative testing strategies with DF and FDF statistics

Following Enders and Lee (2012b), we consider the DF-type regression

$$y_t = d(t) + \rho y_{t-1} + \varepsilon_t, \quad t = 1, \dots, T, \tag{1}$$

where $d(t)$ is the deterministic trend function and ε_t is the stationary disturbance term. We are interested in testing the null hypothesis of a unit root ($H_0 : \rho = 1$) against the alternative hypothesis of stationarity ($H_1 : \rho < 1$). In the linear case, we assume that $d(t) = \delta_0 + \delta_1 t$ (a linear trend), and the usual linear DF test is applied (below, we allow for cases where $\delta_1 = 0$ as well as $\delta_1 \neq 0$). By contrast, if the trend function is nonlinear (either with break(s) or other types of nonlinearity), following Enders and Lee (2012a,b), the trend function is approximated by the Fourier expansion

$$d(t) = \delta_0 + \delta_1 t + \alpha_k \sin(2\pi kt/T) + \beta_k \cos(2\pi kt/T), \tag{2}$$

where k is the Fourier frequency. While the Fourier expansion can be modelled with multiple/cumulative frequencies, given that a Fourier component with a suitable single frequency generally approximates nonlinearity well (Enders and Lee, 2012a,b), throughout the paper we only consider a single-frequency Fourier function. Enders and Lee (2012b, p. 198) warn that "an over-fitting phenomenon occurs when a large number of frequency components are included in the estimating equation" so that the power of the test diminishes rapidly. Nevertheless, for completeness, research into the properties of tests which incorporate multiple/cumulative frequencies is ongoing.

Upon (2) being substituted into (1), the regression model becomes

$$\begin{aligned} y_t &= \delta_0 + \delta_1 t + \alpha_k \sin(2\pi kt/T) \\ &\quad + \beta_k \cos(2\pi kt/T) + \rho y_{t-1} + \varepsilon_t. \end{aligned} \tag{3}$$

We denote the modified DF unit root test based on (3) as FDF(k) where k is, as noted above, the Fourier frequency.

Since the frequency (k) that provides the best fit to the nonlinear trend is unknown, Enders and Lee (2012a,b) suggested a grid search—specifically, selecting $\hat{k}$ over $1 \le k \le 5$ that minimizes the sum of squared residuals from Eq. (3). The resulting test is denoted as FDF($\hat{k}$). Further, Enders and Lee (2012b) suggested pretesting for nonlinearity via the usual F-statistic for the null hypothesis of linearity (i.e., $\alpha_k = \beta_k = 0$) in (3) to determine which of the two unit root tests, DF or FDF($\hat{k}$), should be used. Allowing for the contingent result of pretesting, Enders and Lee (2012b) suggested the following pretesting strategy.

Strategy 1 (*EL Pretesting Strategy*)**.** *If the null of linearity is rejected, perform* FDF($\hat{k}$)*; otherwise, apply the DF test.*

Table 2
Finite sample size ($\rho = 1$) with $T = 200$ at 5% significance level.

k	α_k	β_k	Trend			Level		
			Pretest	UR	Hybrid	Pretest	UR	Hybrid
	0	0	0.078	0.050	0.064	0.074	0.051	0.063
1	0	3	0.064	0.040	0.058	0.071	0.042	0.059
	3	0	0.069	0.042	0.058	0.074	0.047	0.063
	0	5	0.053	0.025	0.049	0.065	0.031	0.054
	3	5	0.052	0.021	0.049	0.050	0.022	0.048
2	0	3	0.045	0.025	0.040	0.054	0.035	0.047
	3	0	0.051	0.027	0.045	0.053	0.033	0.047
	0	5	0.044	0.016	0.043	0.061	0.030	0.053
	3	5	0.046	0.015	0.046	0.053	0.025	0.050
3	0	3	0.039	0.020	0.037	0.057	0.032	0.047
	3	0	0.041	0.022	0.039	0.045	0.027	0.042
	0	5	0.051	0.018	0.050	0.054	0.029	0.051
	3	5	0.048	0.016	0.048	0.053	0.023	0.052

Enders and Lee (2012b) showed via simulation that the above pretesting procedure is useful. However, the procedure tends to be over-sized in some cases and is subject to power loss. In this paper, therefore, we suggest two alternative testing strategies. First, following Harvey et al. (2009), we consider the following UR decision rule.

Strategy 2 (*UR Strategy*)**.** *Reject H0 if either* $\{\mathrm{DF} < \tau^\lambda cv^\lambda_{\mathrm{DF}}\}$ *or* $\{\mathrm{FDF}(\hat{k}) < \tau^\lambda cv^\lambda_{\mathrm{FDF}(\hat{k})}\}$.

In Strategy 2, λ is a given significance level, τ^λ is a scaling constant serving for size adjustment, and cv^λ_{DF} and $cv^\lambda_{\mathrm{FDF}(\hat{k})}$ are critical values of DF and FDF($\hat{k}$), respectively. Following Harvey et al. (2009, rejoinder), the UR strategy can also be represented as

$$\mathrm{UR} = \min\left\{\mathrm{DF}, \left(\frac{cv^\lambda_{\mathrm{DF}}}{cv^\lambda_{\mathrm{FDF}(\hat{k})}}\right)\mathrm{FDF}(\hat{k})\right\}. \tag{4}$$

H_0 is rejected at λ if $\mathrm{UR} < \tau^\lambda cv^\lambda_{\mathrm{DF}} \equiv cv^\lambda_{\mathrm{UR}}$. The UR critical value, cv^λ_{UR}, is obtained via simulation using GAUSS with 100,000 replications and tabulated in Table 1. We note that, as expected, $|cv^\lambda_{\mathrm{UR}}| > |cv^\lambda_{\mathrm{DF}}|$; i.e., the size-correcting parameter τ^λ is larger than 1. In addition, it is necessary to allow for cases where $\delta_1 = 0$, i.e., where there is no deterministic time trend in (3). A separate set of critical values is required for these cases—they are included in Table 1. For convenience, we shall call the case with a time trend the "Trend" case, and the case without a trend, the "Level" case.

Second, we consider a "hybrid" approach suggested by Harvey et al. (2012): combining the UR strategy above with the extra information obtained from the pretesting statistic. The hybrid strategy entails the following.

Strategy 3 (*Hybrid Strategy*)**.** *If the null of linearity is rejected, perform* FDF($\hat{k}$)*; otherwise, apply the UR strategy (Strategy 2).*

It can be seen that Strategy 3 is the same as Strategy 1 when nonlinearity is evident; however, when linearity cannot be rejected, the strategy recommends the use of the UR strategy instead of the DF test. This hybrid approach is expected to achieve a gain in power compared to the pure-UR strategy, by utilizing additional information from pretesting.