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A note on the life-cycle search and matching model with segmented labor markets

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HIGHLIGHTS

- A recent literature develops search and matching models with life-cycle features.
- This note explores a version of such models with age-segmented labor markets.
- Conditions for unambiguous age profiles of job creation and destruction are examined.
- These conditions are compared with those in models with a single labor market.
- Implications for the efficient allocations follow by imposing the Hosios condition.

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ABSTRACT

Recently, there has been renewed interest in labor search and matching models that incorporate a life-cycle structure by assuming finite horizons. Existing studies provide detailed analyses on the age dynamics of job creation and destruction, assuming that workers of all ages search for jobs in the same market. This paper examines a related environment that has drawn less attention, where the labor market is exogenously segmented by age. The paper finds sufficient conditions for the model to yield unambiguous predictions on the age profiles of key variables, and compares them with the corresponding conditions in models with a single market. The paper further examines the age profiles of these key variables in the efficient allocation. In particular, with no persistence in idiosyncratic match productivity, the efficient allocation is found to exhibit monotonic age profiles for the job finding and separation rates.

1. Introduction

Age is an important element impacting the status of workers in the labor market, as observed in, for example, the problem of youth unemployment. Recent literature extends the standard Mortensen–Pissarides model (Mortensen and Pissarides, 1994) by assuming finite working life, to address such age related issues.[1] In particular, Chéron et al. (2011, 2013) provide detailed analyses of the theoretical properties of such models, assuming that workers of all ages search for jobs in a single market, and derive predictions on the age dynamics of job creation and destruction. Understanding the implications of introducing a finite horizon, as well as underlying mechanisms, is important in developing models that both qualitatively and quantitatively account for the observed age patterns of labor market transitions.

This note examines the age dynamics of job creation and destruction in a related environment, in which the labor market is exogenously segmented by age. Few studies adopt such models because of, among other considerations, the presence of age discrimination laws in developed countries; as a result, the life-cycle properties of these models have not been fully examined. The only attempt I am aware of is Chéron et al. (2007),[2] which focuses on an environment in which idiosyncratic match productivity is drawn every period from a uniform distribution. Moreover, the obtained condition leading to unambiguous age profiles of key variables is a complex function of model parameters, with

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1 Such studies include Hahn (2009), Chéron et al. (2011, 2013), and Esteban-Pretel and Fujimoto (2010).

2 Menzio et al. (2010) develops a life-cycle search model in which the labor market is segmented by age, but the model assumes that firms post long-term contracts, which differs substantially from the Mortensen–Pissarides-type models discussed here.

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no straightforward interpretation. Theoretically, the assumption of age-segmented markets complicates the analysis relative to a single market, since age-dependent market tightness provides an additional channel through which age affects the worker's outside options—hence, the match surplus. Besides the consideration of legal restrictions, such a technical difficulty may be another reason why the literature has favored models with a single labor market.

There are several reasons, however, why the analysis of an environment with exogenously segmented markets is valuable. First, whether a single market or age-segmented markets better approximates reality may differ across countries, due to differences in legal restrictions and social norms. Second, the environment provides a useful efficiency benchmark, because the social planner can achieve more efficient allocations by segmenting the market, if that is possible,[3] and moreover, as argued by Chéron et al. (2008, 2011, 2013), the market equilibrium achieves the planner's allocation under the Hosios (1990) condition, if and only if the market is segmented by age.

To address the gap in the literature, this note examines the life-cycle properties of key labor market variables, allowing idiosyncratic match productivity to have general distributions and persistence as in Chéron et al. (2011, 2013),[4] but with age-segmented markets. Remarkably, age-dependent market tightness enables the market equilibrium to generate rich age patterns of job creation and destruction, as in Chéron et al. (2013), which features endogenous search effort.[5] The implications for the efficient allocation also follow by imposing the Hosios condition. In the special case of no persistence in idiosyncratic match productivity, the efficient allocation is found to yield monotonic age profiles of job finding and separation rates.

2. Model

2.1. General environment

The model follows that of Chéron et al. (2011), except that the labor market is exogenously segmented by age. Time is discrete and goes to infinity. The economy is populated by a large number of firms and workers, both of which are risk neutral and discount the future with $\beta \in (0, 1)$. Firms are ex ante homogeneous, and can freely post vacancies by paying a flow cost $c > 0$.

Workers age deterministically, and remain in the labor market for $T - 1$ periods. Workers begin searching for jobs at age $a = 0$, enter the labor market at $a = 1$, and retire at $a = T$. There is a continuum of workers of each age, and the measure of workers of each age is normalized to 1.

Production is performed by pairs of firms and workers, who meet in the labor market, which is exogenously segmented by age. Let market a imply the submarket in which unemployed workers of age a search for jobs. There are $T - 1$ of these submarkets, since workers of age $a = 0$ to $T - 2$ search for jobs. In each submarket, firms and workers are randomly matched according to matching function $M(u_a, v_a)$, where u_a and v_a are, respectively, the number of unemployed workers and vacancies in market a. It is assumed that $u_0 = 1$, whereas for other ages, u_a is determined in equilibrium. The matching function is increasing[6] and continuously differentiable in both arguments, is concave, and exhibits CRS. Letting $\theta_a \equiv v_a/u_a$ be the tightness of market a, a firm finds a worker with probability $q(\theta_a) \equiv M(u_a, v_a)/v_a$, and a worker finds a firm with probability $p(\theta_a) \equiv M(u_a, v_a)/u_a = \theta_a q(\theta_a)$. I focus attention on the case in which all $T - 1$ submarkets operate and there is no need for truncating the matching function. That is, $\theta_a \in \Theta$ for all a, where $p(\theta), q(\theta) \in (0, 1)$ for all $\theta \in \Theta$.[7]

The flow output of a match is $\epsilon \in [\epsilon_{\min}, \epsilon_{\max}]$, where productivity ϵ is idiosyncratic to each match. When a match is formed, the initial value of ϵ is drawn from a distribution with cumulative density function $G(\epsilon)$. In subsequent periods, with probability $\lambda \in [0, 1]$, a new value of ϵ is drawn from $G(\epsilon)$, whereas with probability $1 - \lambda$, ϵ remains unchanged.

Matches terminate from two sources. First, a match is destroyed endogenously if ϵ is so low that the match surplus is negative. A newly formed match may also face such endogenous destruction if initial ϵ is sufficiently low. Second, a match terminates when the worker turns age T and retires.

While employed, workers receive wages, which are determined each period through Nash bargaining. While unemployed, workers receive the flow value of leisure, $b > 0$.

2.2. Equilibrium

Let us turn to the stationary equilibrium, beginning with the value functions of the firm and the worker. Since workers retire at age T, the value functions involving workers of age $T - 1$ differ from those for other ages.

Let $J_a(\epsilon)$ be the firm's value of a filled job with a worker of age a. Then,

$$J_{T-1}(\epsilon) = \epsilon - w_{T-1}(\epsilon), \tag{1}$$

$$\begin{aligned} J_a(\epsilon) = {} & \epsilon - w_a(\epsilon) + \beta(1-\lambda)\max\{J_{a+1}(\epsilon) - V, 0\} \\ & + \beta\lambda\int_{\epsilon_{\min}}^{\epsilon_{\max}} \max\left\{J_{a+1}(\epsilon') - V, 0\right\} dG(\epsilon') \\ & + \beta V, \quad a = \{1, 2, \ldots, T-2\}, \end{aligned} \tag{2}$$

where $w_a(\epsilon)$ is the wage paid to the worker. The firm's value of posting a vacancy is $V = \max V_a$, $a = \{0, 1, \ldots, T - 2\}$, where V_a is the value of posting a vacancy in market a, expressed as

$$V_a = -c + \beta q(\theta_a)\int_{\epsilon_{\min}}^{\epsilon_{\max}} \max\left\{J_{a+1}(\epsilon') - V, 0\right\} dG(\epsilon') + \beta V. \tag{3}$$

Let $W_a(\epsilon)$ be the value of being employed for a worker of age a. Then,

$$W_{T-1}(\epsilon) = w_{T-1}(\epsilon), \tag{4}$$

$$\begin{aligned} W_a(\epsilon) = {} & w_a(\epsilon) + \beta(1-\lambda)\max\{W_{a+1}(\epsilon) - U_{a+1}, 0\} \\ & + \beta\lambda\int_{\epsilon_{\min}}^{\epsilon_{\max}} \max\left\{W_{a+1}(\epsilon') - U_{a+1}, 0\right\} dG(\epsilon') \\ & + \beta U_{a+1}, \quad a = \{1, 2, \ldots, T-2\}. \end{aligned} \tag{5}$$

Here, U_a is the value of being unemployed for a worker of age a, expressed as

$$U_{T-1} = b, \tag{6}$$

$$\begin{aligned} U_a = {} & b + \beta p(\theta_a)\int_{\epsilon_{\min}}^{\epsilon_{\max}} \max\left\{W_{a+1}(\epsilon') - U_{a+1}, 0\right\} dG(\epsilon') \\ & + \beta U_{a+1}, \quad a = \{1, 2, \ldots, T-2\}. \end{aligned} \tag{7}$$

[3] Given the constant returns to scale (CRS) matching function generally assumed, the planner can never do worse, and in general can do strictly better, by choosing segmented markets, since they provide the additional freedom of making market tightness dependent on age.

[4] Chéron et al. (2011) discusses the case of persistent productivity as an extension.

[5] Due to endogenous search effort, the model of Chéron et al. (2013) yields richer age dynamics of job creation and destruction than that of Chéron et al. (2011).

[6] Throughout, increasing (decreasing) implies strictly increasing (decreasing), and nondecreasing (nonincreasing) implies weakly increasing (decreasing).

[7] While the potential need for truncation arises in any discrete time search and matching model, the issue is more relevant in this environment, in which tightness may vary substantially across markets.