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Power monotonicity in detecting volatility levels change☆

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HIGHLIGHTS

- We show that the CUSUM and LM tests for structural change in volatility enjoys monotonic power.
- The traditional results regarding non-monotonic power for changing mean do not apply in our context.
- Our specification of structural change is general.
- Simulations and an application provide further support.

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ABSTRACT

We show that the CUSUM and LM tests for structural change in the volatility process enjoy monotonic power. The framework is general including many recently proposed non-stationary GARCH-type models. The result is in contrast to the well-known issue of non-monotonic power for the CUSUM-based tests for changing mean. Simulations and an empirical example provide further support.

1. Introduction

Since the successful capturing of volatility clusterings in asset returns by Engle and Bollerslev's autoregressive conditional heteroskedastic model, it has been long recognized that many other stylized features in volatilities can be well explained if the model additionally incorporates a structural change. These features include long range dependence, IGARCH effects, and dominance of very low frequencies in the periodogram; see Diebold (1986), Mikosch and Stărică (2004), Stărică and Granger (2005), Hillebrand (2005) and Perron and Qu (2010), *inter alia*. Smooth change in volatility levels is recently considered by Engle and Rangel (2008), Engle et al. (2009), and Amado and Teräsvirta (2012) in their volatility components models. In the regression context, checking non-stationary volatility also helps to determine whether more sophisticated statistical procedures are necessary (Cavaliere and Taylor, 2007; Xu and Phillips, 2008; Xu, 2012).

The CUSUM and other related tests (constructed with squared series) are routinely implemented to test for changing volatility levels (Aggarwal et al., 1999, Andreou and Ghysels, 2002, Rapach and Strauss, 2008, among many others). This test was first considered by Inclán and Tiao (1994) in an i.i.d. setting, and modifications that allowed for serial correlation and dynamic conditional heteroscedasticity in squared returns were proposed by Loretan and Phillips (1994), Andreou and Ghysels (2002), Sansó et al. (2004), and Deng and Perron (2008b). In this paper we focus on the power of the CUSUM test. The CUSUM test, despite its simplicity in implementation and the non-parametric nature, has the main drawback of possibly having non-monotonic power when it is applied to test for *changing mean* (Vogelsang, 1999; Deng and Perron, 2008a; Juhl and Xiao, 2009). Although the close connection between two

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CUSUM tests (mean change test and volatility change test) has been noted (they have the same distribution under the null), the behavior of the *volatility change test* under alternatives especially the shape of the power function, however, does not follow from the *mean change test*. In Section 2, we present the model and the results. The connection between two types of tests is established and the contrasts of the results are emphasized. Monotonic power of the volatility change test is explained by the fact that a structural change in volatility forces the same structural change to be imposed on the volatility of volatility, thereby preventing the automatic procedure selecting too many lags in the long-run kurtosis estimator, the main potential source of non-monotonic power. Numerical illustration is presented in Section 3. Technicalities are contained in an Appendix.

2. The model and the results

Consider the following model for a martingale difference sequence $R_t = u_t$ (e.g. log returns):

$$u_t = \sigma_t \varepsilon_t, \tag{1}$$

where $t = 1, \ldots, n$, σ_t is deterministic and ε_t is stationary such that $E(\varepsilon_t|\mathcal{I}_{t-1}) = 0$, $E\varepsilon_t^2 = 1$.[1] The series $\varepsilon_t^2 - 1$ has autocovariances γ_l $(l = 0, 1, 2, \ldots)$ such that the finite long-run variance (LRV) $\lambda^2 = \gamma_0 + 2\sum_{l=1}^{\infty}\gamma_l < \infty$. The unconditional variance process σ_t^2 is generated by a non-stochastic cádlág function $\sigma_t^2 = \sigma^2(t/n)$, where $\sigma^2(\cdot)$ is twice differentiable except at a finite number of points of discontinuity on $(0, 1]$ with the second derivative function satisfying a first-order Lipschitz condition piecewise. The process ε_t captures dynamic conditional heteroskedasticity, usually by a stationary GARCH-type process. Engle and Rangel (2008) interpreted σ_t^2 as the low-frequency component of volatility that reflects the long-run dynamics of the volatility process.

The hypothesis of interest is $H_0 : \sigma_t^2 \equiv \sigma^2$ versus $H_A : \sigma_t^2$ is not constant over t. Several parametric non-stationary volatility models exist in the literature, e.g. σ_t^2 as a spline function in Engle and Rangel (2008), a smooth transition logistic function in Amado and Teräsvirta (2012), and a step function in Mikosch and Stărică (2004) and Stărică and Granger (2005).

Let $\widehat{e}_t = u_t^2 - n^{-1}\sum_{t=1}^{n}u_t^2$. In this paper we focus on the non-parametric test based on the cumulative sum process (the CUSUM test) defined as $Q = \max_{1\le K\le n} Q(K)$, where $Q(K) = |n^{-1/2}\sum_{t=1}^{K}\widehat{e}_t|/\widehat{\omega}$ and $\widehat{\omega}^2 = \widehat{\varphi}_0 + 2\sum_{l=1}^{n-1}k(l/m)\widehat{\varphi}_l$ with $\widehat{\varphi}_l = n^{-1}\sum_{t=l+1}^{n}\widehat{e}_t\widehat{e}_{t-l}$. Here $k(\cdot)$ is the kernel function assigning weights to autocovariances in the LRV estimator $\widehat{\omega}^2$. The truncation parameter m is usually selected data-dependently involving a preliminary parametric fit. We here consider the criterion $m = 1.1447[4(1-\widehat{\rho})^{-2}(1+\widehat{\rho})^{-2}\widehat{\rho}^2 n]^{1/3}$, where $\widehat{\rho} = \widehat{\varphi}_1/\widehat{\varphi}_0$, proposed by Andrews (1991), and it is optimal for the LRV estimation when Bartlett kernel $k(x) = (1-|x|)\mathbb{I}_{\{|x|\le 1\}}$ is used.[2] The asymptotic distribution of the Q test statistic under the null hypothesis can be found in Inclán and Tiao (1994), and the critical values are 1.224 (10%), 1.358 (5%), and 1.628 (1%).

We consider the following specification of the alternative hypothesis allowing for local and non-local deviations from the null

$$\sigma_t^2 = \sigma^2(t/n) = \sigma_0^2 + g(t/n)\xi, \tag{2}$$

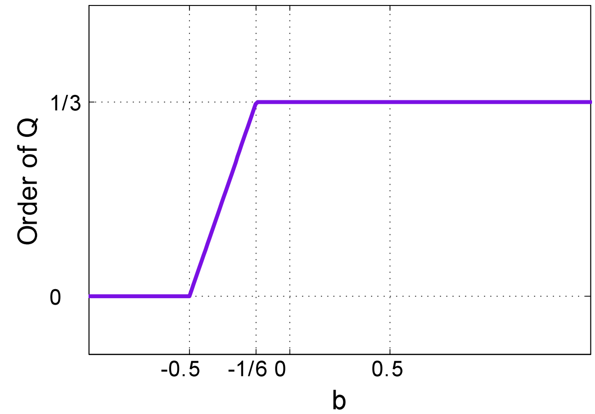

Fig. 1. The stochastic order of Q (i.e. d, where $Q = O_p(n^d)$) versus b.

where σ_0^2 is a constant, $\xi = n^b$ with $-\infty \le b < \infty$, and $g(\cdot)$ is a bounded function. The case $b = -\infty$ corresponds to the null. The local alternative ($b < 0$) is more traditional for power analysis in the spirit of the Pitman drift, and the non-local alternative is convenient for the examination of (non-)monotonic shape of the power function (Juhl and Xiao, 2009). In the case when there is a single break in variance from σ_0^2 to $\sigma_\dagger^2$ at time $\lfloor n\tau \rfloor$, $\sigma^2(r) = \sigma_0^2 + (\sigma_\dagger^2 - \sigma_0^2)\mathbb{I}_{\{r\ge\tau\}}$, $r \in (0, 1]$, and in the specification of (2), $\xi = \sigma_\dagger^2 - \sigma_0^2$ and $g(r) = \mathbb{I}_{\{r\ge\tau\}}$. The following theorem characterizes the stochastic order of Q under H_A.

Theorem 1. *Under the stated assumptions and H_A, where σ_t^2 assumes the form in (2), $Q = O_p((1+n^{1/3+2b})^{-1/2}(1+n^{1/2+b}))$. The stochastic order of Q against b is plotted in Fig. 1.*

Theorem 1 shows that the test is consistent if $b > -1/2$, and $Q = O_p(n^{1/3})$ if $b \ge -1/6$.

Juhl and Xiao (2009) studied the CUSUM test for changing mean. They showed that the test has non-monotonic power and it is not even consistent when the change is too large. The main source of such inconsistency is that under a large mean change, the AR coefficient estimate $\widehat{\rho}$ is seriously biased toward unity so that the data-dependent procedure selects a too large m (of order up to $O_p(n)$). Our test is closely related to Juhl and Xiao's. Indeed, the statistic Q is the application of their test statistic to u_t^2. Rewrite the model (1) as

$$u_t^2 = \sigma_t^2 + e_t \tag{3}$$

where $e_t = \sigma_t^2(\varepsilon_t^2 - 1)$. The issue of non-monotonic power, however, does not apply to Q as the results of Juhl and Xiao strongly depend on the assumption that the variance is a bounded constant and is unassociated with the mean, which is not satisfied here. Moreover, (3) implies that under the alternative the same structural change occurs in both the volatility and the volatility of volatility.[3,4]

In fact, under H_A, we can show that for a large volatility change such that $b > 1/2$,

$$\operatorname*{plim}_{n\to\infty}\widehat{\rho} = \frac{\gamma_1 + \left[1 - \left(\int g\right)^2/\int g^2\right]}{\gamma_0 + \left[1 - \left(\int g\right)^2/\int g^2\right]} \ne 1; \tag{4}$$

[1] We only consider zero-mean models as in canonical GARCH-type models, and the results can be extended to models with the correctly specified conditional mean function and the CUSUM and LM tests that are modified correspondingly. When the mean is suspected to be misspecified, the OLS-based tests are general invalid and non-parametric methods (with u_t replaced by non-parametric regression residuals) will be useful.

[2] The main results in this paper extend to other kernels in which cases m could be of a different order of n.

[3] It is worth mentioning that Q is conceptually different from the CUSUM of squares test originally proposed by Brown et al. (1975) since here we are mainly interested in *volatility* change (and assume no structural change in mean). Following the line of Brown et al., Deng and Perron (2008a) studied the behavior of the CUSUM of squares test to detect the change in *mean* assuming constant bounded variance, and they found the test has non-monotonic power.

[4] The size and power properties of the CUSUM test in Juhl and Xiao (2009) under general heteroskedastic errors are analyzed in Xu (2011).