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An impossibility result for virtual implementation with status quo

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HIGHLIGHTS

- We study the implementation of social choice functions (SCFs) on unrestricted domain.
- We use virtual implementation mechanisms restricted to a fixed outcome (a status quo).
- On the unrestricted domain, unanimity and monotonicity lead to nearly dictatorial SCFs.

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ABSTRACT

In this paper, virtual implementation is restricted to deliver, on the equilibrium path, either a socially optimal outcome or a *status quo*: an outcome fixed for all preference profiles. Under such a restriction, for any unanimous and implementable social choice function there is a dictator, who obtains her most preferable outcome as long as all agents prefer this outcome to the status quo. Further restrictions on the lottery space and the range of social choice functions allow the dictator to impose her most preferred outcome even when other agents prefer the status quo to this outcome.

1. Introduction

Suppose that a society is interested in devising a mechanism to implement a rule that prescribes a single socially optimal outcome in each state of the world. We call such a rule a social choice function (SCF). If an SCF needs to cover all possible states of the world, the following impossibility result arises: with at least three alternatives, an SCF that is unanimous and exactly implementable has to be dictatorial. However, if the designer is allowed to implement an SCF approximately, any SCF is implementable.

This paper studies a virtual implementation problem when the designer is restricted on how she can approximate an SCF: instead of using an arbitrary outcome, she is restricted to using a single fixed outcome, which we call *status quo*. Under this restriction, unanimity and implementability again imply a form of dictatorship.

The key condition for exact implementation is Maskin-monotonicity (Maskin, 1999). Suppose that we are interested in implementing an SCF that prescribes an alternative for every possible profile of preferences (unrestricted domain of preferences); in such a case, the only implementable SCFs are those that prescribe the same alternative for every preference profile (Saijo, 1987). This constancy result relies on the profiles in which an agent is indifferent to the alternatives. If only strict preferences are considered, some non-constant SCFs can also be implemented. Suppose that, in addition to Maskin-monotonicity, we require unanimity: whenever all agents agree on the most desirable outcome, an SCF chooses that outcome. SCFs that are unanimous and Maskin-monotonic are once again restricted to a narrow class of dictatorial SCFs (Muller and Satterthwaite, 1977).[1] These are the SCFs in which the preferences of one individual (a dictator) fully determine social outcomes.

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[1] This work builds upon the pioneering papers by Gibbard (1973) and Satterthwaite (1975).

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One way to avoid this negative result is to introduce a mild relaxation of the implementability requirement. Suppose that society is willing to tolerate a mechanism that delivers, on the equilibrium path, a non-socially optimal outcome with small probability. This relaxation, called virtual implementation, is extremely powerful in evading the restriction to dictatorial rules: it allows the implementation of nearly every SCF (Abreu and Sen, 1991; Matsushima, 1988). However, it comes with a caveat: it places no restrictions on what non-socially optimal outcomes can be delivered on the equilibrium path. It is possible that some of these outcomes are socially intolerable.

A natural response to this problem is to restrict the type of outcomes that can be delivered on the equilibrium path. Bochet and Maniquet (2010) study this problem. They call the outcomes that are admissible as equilibrium outcomes an "admissible support" (an admissible outcome could be different for two different preference profiles). This formulation includes both exact implementation and virtual implementation. To obtain the former, one restricts an admissible support to the socially optimal outcome only. To obtain the latter, an admissible support includes every alternative.

Bochet and Maniquet (2010) derive a necessary and sufficient condition for virtual implementation with admissible support and provide several examples of correspondences that are not exactly implementable, but virtually implementable with a natural (and small) admissible support. They further show how to construct, for any given SCF, a minimal admissible support that allows the implementation of this SCF.

In this paper, we examine virtual implementation with admissible support where the support is restricted to two outcomes only: a socially optimal outcome and a fixed outcome, which we interpret as a status quo. That is, on the equilibrium path, the mechanism can either "succeed", with large probability, and result in a socially optimal outcome, or "fail", with small probability, and result in the status quo. We call this virtual implementation with status quo.

We show that virtual implementation with status quo escapes the constancy result of Saijo (1987), even though we allow for indifferences. Yet, the negative result of Muller and Satterthwaite (1977) partially extends to our notion of implementation. We show that the dictator is able to impose her preferences on society as long as everyone is better off with her choice than with the status quo.

Although we are using the framework of Bochet and Maniquet (2010), we are not able to exploit the characterization that they obtain for implementation with admissible support. The reason for this is that Bochet and Maniquet (2010) do not assume expected utility (assuming instead the much weaker notion of monotonicity in probabilities) and, without loss of generality, restrict their attention to ordinal mechanisms. Since we assume expected utility in this paper, the restriction to ordinal mechanism is untenable (e.g., see a discussion after Lemma 3). Therefore, we re-derive the necessary and sufficient condition. By doing so, we are effectively applying insights from Benoit and Ok (2008), Bochet (2007) and Sanver (2006) to the present paper.

Some of the proof techniques of this paper are adopted from Serrano (2004), who, in turn, adopts the proofs of Reny (2001) and Mas-Colell et al. (1995). However, as the domain of preferences considered in this paper is restricted by the assumption that preferences over lotteries can be represented by expected utility, the proofs of this paper are not implied by this earlier work. The impossibility results have been extended to lottery space in Barberà et al. (1998) and Benoit (2002), but both papers consider problems different from ours.

In the next section, we present the notation and state properties of an SCF. In Section 3, we derive a monotonicity condition relevant for the present paper. In Section 4, we discuss the constancy result. In Section 5, we collect supplementary lemmas. In Section 6, we show our main result that monotonicity and unanimity imply a q-constrained dictatorship. In Section 7, we outline a stronger result when the lottery space and the range of SCFs are further restricted. We conclude in Section 8.

2. Preliminaries

Let $A \cup \{q\}$ be the finite set of outcomes, where q is an outcome designated as the status quo. Let $|A| = k$. Let N be the finite set of agents, and $|N| = n$.

Let the lottery space be $\mathcal{L} = \Delta(A \cup \{q\})$, where Δ is the probability simplex. We assume that preferences over lotteries satisfy standard assumptions (completeness, transitivity, continuity, and independence), so that they allow the standard von Neumann–Morgenstern expected utility representation.

Denote the set of possible profiles by Θ, with $\Theta \subseteq \Re^{k\times n}$; a generic element of this set is denoted by θ. For each $\theta \in \Theta$ and for each $i \in N$, we normalize the utility from q to be $u_i(q;\theta) = 0$. A matrix corresponding to each θ is the collection of the vectors of utilities of agent $i \in N$ for outcomes $(a_1, \dots, a_k)$. Note that, as we only fix the utility from q, two proportional vectors represent the same preferences.

We define two domains of preference profiles. First, let $\Pi^{++} = (\Re_{++})^{k\times n}$. For each preference profile $\theta \in \Pi^{++}$, the utility from any $a \in A$ is higher than the utility from q. Second, let $\Pi^{+} = \{\theta \in \Re^{k\times n} | \forall i \in N \exists a \in A : u_i(a;\theta) > 0\}$. For each $\theta \in \Pi^{+}$, there is at least one outcome that is better than q for every agent.

For each agent $i \in N$ and each $\theta \in \Theta$, we denote the strictly worst outcome from the set A for this agent by $a_{(1;i,\theta)}$ and the strictly best outcome from A by $a_{(k;i,\theta)}$. If the agent has several (weakly) best outcomes, we denote the set of such outcomes by $A_{(k;i,\theta)}$.

An SCF f is a mapping $f : \Theta \mapsto (A \cup \{q\}) \times (0, 1]$. Note that an SCF can only prescribe an outcome that is a lottery involving an outcome $a \in A$ and the fixed outcome q; no lotteries that involve outcomes $a, b \in A$ are allowed in the range of an SCF. For compactness, we will write a lottery that assigns probability p to outcome a and, correspondingly, probability $(1 - p)$ to the status quo q by (a, p); further, whenever a lottery assigns probability 1 to a, we will denote such a lottery by a. As we normalize the utility from the status quo to 0, the expression for expected utility of a lottery (a, p) becomes $u_i((a, p);\theta) = pu_i(a;\theta)$.

We now define several properties of an SCF.

Definition 1. An SCF f is *dictatorial* if there exists an agent $j \in N$, such that for any $\theta \in \Theta$, $f(\theta) \in A_{(k;j,\theta)}$.

Definition 2. An SCF f is *q-constrained dictatorial* if there exists an agent $j \in N$, such that for any $\theta \in \Theta$, which satisfies the condition that for all $a \in A_{(k;j,\theta)}$ and all $i \in N$ $u_i(a;\theta) > 0$, it follows that $f(\theta) \in A_{(k;j,\theta)}$.

That is, for a q-constrained dictatorial function, the dictator is able to impose her preferred outcome only when this outcome is better than the status quo.

Definition 3. An SCF f is *unanimous* if, whenever for any $\theta \in \Theta$, there exists $a \in A \cup \{q\}$, such that $u_i(a;\theta) > u_i(b;\theta)$ for all $i \in N$ and all $b \in A \cup \{q\} \setminus \{a\}$, it follows that $f(\theta) = a$.

3. A monotonicity condition

Bochet and Maniquet (2010) introduce a notion of virtually Nash implementable SCF with admissible support, which we use in