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The stability of decision making in committees: The one-core

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HIGHLIGHTS

- Study the stability of decision making in committees with endogenous policy proposals.
- Provide a necessary and sufficient condition for the existence of a stable policy outcome.
- Show trade-off between the size of a committee, the number of competing policy options, and the existence of a stable outcome.

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ABSTRACT

We study the stability of decision making in committees. A policy proposal introduced by a committee member is either adopted or abandoned in favor of a new proposal after deliberations. If a proposal is abandoned, it is in spite of the committee member who introduced it, who does not cooperate in any effort to defeat it. Shenoy (1980) proposes the one-core as a solution concept for this game, and shows that this solution may be empty. We provide a necessary and sufficient condition for the existence of a stable policy under the majority rule. This result highlights a trade-off between the size of a committee, the number of competing policy options, and the existence of a stable outcome. Our findings imply a tension between political stability and the existence of a large number of competing interests in democracies.

1. Introduction

We study the stability of decision making in a committee in which a policy proposal introduced by a member is either adopted or abandoned in favor of a new proposal after deliberations. If a proposal is abandoned, the committee member who introduced this proposal does not cooperate in any effort to defeat it.

This committee game was introduced by Shenoy (1980) to model decision making in certain real-world contexts. We formalize this game as the tuple (N, A, v, R) where N is a finite group of individuals who have to choose one alternative from a finite set of alternative policies A; v is the majority rule, which is the rule by which the committee arrives at a decision[1]; and R is a preference profile over the set of policy alternatives A.

Consider a decision-making process which begins when a committee member i introduces an alternative a_1 in the form of a proposal (i, a_1). Next, one of two situations arises: either the committee agrees on the adoption of a_1 and the game ends, or another committee member $j \in N$ introduces a motion a_2 in the form of a proposal (j, a_2). The proposal is then debated by the members and at the end of the debate one of two possibilities occurs. The first is that there exists a winning coalition S that asks for the adoption of a_2, in which case the game is over and the final decision is a_2. If such a coalition exists, it cannot include i ($i \notin S$), as i is not allowed to cooperate in any effort to defeat his own proposal a_1. The second

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1 In the original model introduced by Shenoy (1980), v is more generally defined, but here we limit our consideration to the majority rule.

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possibility is that a winning coalition in favor of a_2 does not exist and player j withdraws his motion; no other motion is introduced, and the final decision is a_1. In the first case where the final decision is a_2, we say that proposal (i, a_1) is defeated by proposal (j, a_2). A proposal is said to be stable if it is never defeated by any other proposal. The solution to the game, called the *one-core*, is the set of all stable proposals.[2]

The one-core is related to the core, but both solution concepts have important differences. The voting rule underlying the core implicitly allows a player who has made a policy proposal to then join an opposing coalition in order to defeat that proposal; this makes little sense in real political contexts. To correct this shortcoming of the core, Shenoy (1980) introduced the one-core, which does not permit the player who has initiated a proposal to belong to an opposing coalition which has formed to defeat it. According to the rational behavior underlying the one-core, a player who is making a proposal therefore only cares about undominated outcomes via coalitions not containing him, and picks only maximal ones. It should also be noted that from a purely mathematical and conceptual point of view, the one-core differs from the core in that it specifies the policy alternative that a player *will propose* if asked to do so, whereas the core is merely a set of policy options.

The goal of a decision-making committee is to select a *stable* outcome. Unfortunately, the one-core, like the core, may be empty. Shenoy (1980) provided a five-player committee game with an empty one-core, but proved that the one-core cannot be empty if the committee size does not exceed four individuals. We complete this study by examining conditions under which the one-core is nonempty when the number of players is greater than four.

Our main finding consists of providing a necessary and sufficient condition under which the one-core is always nonempty under the majority rule. For a given committee size, it identifies the maximal number of alternative policies that can be allowed to compete in order for stability to be attained (Theorem 1). This result finds a trade-off between the size of a voting committee, the number of competing policy options, and the existence of a stable outcome.

Our analysis is related to previous studies examining the nonemptiness of the core of voting games (Peleg, 1978; Nakamura, 1979; Kumabe and Mihara, 2011), but the findings differ significantly. More generally, the maximal number of policy options which ensures for the existence of a stable outcome is greater for the one-core than for the core. Under the majority rule, it is well-known that the core may be empty when the number of policy alternatives exceeds two, the only exception being when there are four players, in which case the number of policy alternatives should not exceed three. In contrast, the one-core is always nonempty regardless of the number of policy alternatives when there are at most four players. This is not the case, however, when there are more than four players. For a stable outcome to exist, there should be at most four policy alternatives if there are five or eight players, at most five policy alternatives if there are six players, at most three policy alternatives if there are seven or ten players, and at most two alternatives when there are nine or at least eleven players. Therefore, it is only in this latter case that our result coincides with that of the core. Clearly, our findings imply a tension between political stability and the existence of a large number of competing interests in democracies.

The rest of the paper is organized as follows. Section 2 is devoted to the model and preliminary definitions. Section 3 presents our main result. Finally, Section 4 concludes the paper.

[2] We note that the initial decision-making framework Shenoy (1980) proposes is sequential, whereas our framework is reminiscent of a one-shot game. The one-core, as a solution concept, is better justified by our framework, as in a sequential framework, players must be farsighted to be rational (e.g., Chwe, 1994).

2. The setting

2.1. Committee

A *committee* is defined as a finite set of players denoted $N = \{1, 2, \ldots, n\}$. Its members have to choose one option from a finite set of *policy options* A. It is assumed that A has at least two elements. Nonempty subsets of N are called *coalitions* and the set of all coalitions of N is denoted by 2^N. For any set T, $|T|$ denotes the cardinality of T. We pose $|N| = n$.

Each player i has a preference relation $\succeq_i$ on A which is a weak order (that is, reflexive and transitive). A profile $R = (\succeq_i)_{i\in N}$ is a collection of individual preferences.

Note that R could also be replaced with a utility vector $u = (u^i)$, where $u^i : A \to \mathbb{R}$ denotes the real-valued ordinal utility function of player i. In this case, utility is assumed to be nontransferable and interpersonal comparison of utilities is meaningless.

The rule by which a committee arrives at a decision is the majority rule, denoted by v. Therefore, the rule v is a mapping from 2^N to $\{0, 1\}$ such that for any coalition S, $v(S) = 1$ (that is, S is a winning coalition) if and only if S has more than $\frac{n}{2}$ individuals. Denote by q the smallest integer larger than $\frac{n}{2}$. Any coalition of size q is said to be a minimal winning coalition. Denote by W the set of all winning coalitions. For any individual $i \in N$, we denote by $W(i) = \{S \in W : i \notin S\}$ the set of winning coalitions to which i does not belong.

The tuple $\Gamma = (N, A, v, R)$ or $\Gamma = (N, A, W, R)$ is called an (ordinal) n-person committee game. The committee aims at choosing one option from the set A of policy alternatives. Because Shenoy (1980) was primarily concerned with very small committees, he considered the members of the committee to be situated in one room. In contrast to Shenoy (1980), we will derive our result for committees of any size.

2.2. The core and the one-core

We recall the definitions of the core and the one-core.

Definition 1. Let $\Gamma = (N, A, v, (\succeq_i)_{i\in N})$ be a committee game.

1. A policy a dominates another policy b via a winning coalition S, denoted by $a\ dom_S\ b$, if $\forall i \in S, a \succ_i b$.
2. A policy a dominates another policy b, denoted by $a\ dom\ b$, if there exists a winning coalition $S \in 2^N$ such that $a\ dom_S\ b$.
3. The core of Γ, denoted by $\mathcal{C}(\Gamma)$, is the set of all undominated policies: $\mathcal{C}(\Gamma) = \{b \in A : not(\exists a \in A, a\ dom\ b)\}$.

It follows from this definition that the rational behavior underlying the core prescribes that a voter should vote for an alternative a against another alternative b whenever he prefers a to b.

The one-core is a solution concept introduced by Shenoy (1980) following a modification of the core based on practical considerations. According to Shenoy (1980), a member of a committee to whom it is given the opportunity to make a proposal should not cooperate in any effort to defeat it. This happens for instance when such an individual has to leave the room to let the remaining members of the committee decide on whether or not to adopt his proposal. The faculty of an economics department, for example, might have to take a vote to decide on whether or not to promote a colleague. The latter usually is absent from all discussions and is not allowed to vote.

It follows that if a committee member cannot vote against his own proposal, he should propose a policy that is maximal with respect to his preference relation, and which cannot be defeated by other committee members forming a winning coalition. Formally, let $P = \{(i, x) : i \in N, x \in A\}$ be the set of all proposals. For each $i \in N$, define:

$\hat{C}^i = \{(i, x) \in P : x \text{ is not dominated via any } S \subseteq N \setminus \{i\}\}$.

$\hat{C}^i$ represents the set of proposals made by i that are undominated assuming player i's noncooperation in any effort to defeat his proposal.