Economics Letters 122 (2014) 396–399

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A nearly optimal auction for an uninformed seller[☆]

Natalia Lazzati [a,*], Matt Van Essen [b]

[a] University of Michigan, Department of Economics, United States

[b] University of Alabama, Department of Economics, Finance, and Legal Studies, United States

HIGHLIGHTS

- The seller does not know the distribution of values of potential buyers.
- Our selling mechanism combines an elicitation mechanism with a standard auction.
- The uninformed seller achieves nearly the same expected revenue as Myerson (1981).

ARTICLE INFO

Article history:
Received 22 October 2013
Received in revised form
11 December 2013
Accepted 20 December 2013
Available online 3 January 2014

JEL classification:
C72
D44
D47
D82

Keywords:
Optimal auction
Probability elicitation
Mechanism design

ABSTRACT

This paper describes a nearly optimal auction mechanism that does not require previous knowledge of the distribution of values of potential buyers. The mechanism we propose builds on the new literature on the elicitation of information from experts. We extend the latter to the case where the secret information shared by the experts – potential buyers in our model – can be used against them if it becomes public knowledge.

1. Introduction

The optimal mechanism to sell a single object requires prior knowledge of the distributions of values of potential buyers.[1] In the symmetric independent private value model, for instance, the optimal direct mechanism can be obtained by using a second price sealed bid auction with a specific reservation price. The reservation price depends on the distribution of bidder values. The construction of an optimal auction for the asymmetric case is similarly tethered to the seller's knowledge of the value distribution of each individual bidder. What if the seller does not know these distributions? We provide an almost optimal mechanism for an uninformed seller in a context where the group of potential buyers are aware of the value distributions. In terms of Krishna (2010), our mechanism has the advantage of being "detail-free".

The nearly optimal auction we propose consists of two mechanisms: an elicitation mechanism and an auction. The aim of the elicitation mechanism is to recover the distributions of values of the potential buyers whereas the aim of the auction is to maximize expected revenue. These two mechanisms are intimately related. The details of the auction depend on the distributions obtained from the bidders in the elicitation mechanism, and the lottery payoffs of the elicitation mechanism depend on the bids placed by the potential buyers in the auction. Despite the informational disadvantage placed on the seller, the auction we propose almost always obtains the maximal expected revenue for the seller at a near zero cost. Moreover, the induced game among potential buyers is individually rational and strict incentive compatible.

The elicitation part of our mechanism builds on the recent literature about information elicitation from experts. Karni (2009) introduces an incentive compatible mechanism for eliciting the subjective probabilities of an agent about a finite number of

☆ We are grateful to Mark Walker and Tilman Börges for comments.

* Corresponding author. Tel.: +1 734 764 7438; fax: +1 734 764 2769.
E-mail addresses: nlazzati@umich.edu (N. Lazzati), mjvanessen@cba.ua.edu (M. Van Essen).

1 See Laffont and Maskin (1980), Myerson (1981), or Riley and Samuelson (1981).

0165-1765/$ – see front matter
http://dx.doi.org/10.1016/j.econlet.2013.12.028

events.[2] Qu (2012) extends Karni's mechanism to the elicitation of an agent's beliefs about the general distribution of a random variable. These papers assume that the expert has no stake in the random behavior of interest.[3] In our model, the information disclosed by the experts – the bidders in the auction – will be used by the seller against them. We show that Karni and Qu's contributions can be extended to this delicate situation whenever there are at least two experts or bidders in our context. In this sense, our approach formalizes the well-known phrase.

A secret between more than two is not a secret.

There are a few other papers related to our idea. The recent literature on the econometrics of optimal auctions solves the problem of the uninformed seller by using a sequential auction mechanism (see, e.g., Paarsch, 1997). That is, this literature assumes that the seller runs (or has run) an initial auction, obtains some data, and then uses this information to recover the value distributions of the bidders. The seller then computes and conducts an optimal *new* auction for subsequent units of the good. This procedure is costly in terms of foregone revenue and may not be practical if the seller has a unique item and cannot therefore take advantage of the obtained information. Segal (2003) addresses the same issue by providing a mechanism that sets a price for each buyer on the basis of the demand distribution inferred statistically from other buyers' bids.[4] The resulting profit converges to the optimal monopoly profit with known demand as the number of buyers goes to infinity. We assume that consumers know the distribution of valuations. Under this condition, the advantage of our approach over the one of Segal is that, in our set-up, profits are almost optimal even when the number of bidders is very small (i.e., two bidders in the symmetric model and three bidders in the asymmetric one). Brooks (2013) has recently considered a similar problem but proposed a very different solution. Our mechanism is simpler and the seller controls (via the lottery prizes) the maximum cost he could incur to recover the valuations of potential consumers. On the other hand, Brooks' mechanism allows for certain type of correlation. The project finally relates to the newer literature on the robust mechanism design (see, e.g., Bergemann and Morris, 2005 and Börges, 2013). The latter builds on the observation that the mechanism design literature assumes too much common knowledge of the environment among the players and planner and aims at relaxing this restriction. We keep common knowledge of the environment among the players but relax the information requirement often imposed on the seller.

2. Almost optimal mechanism for the symmetric model

We model a situation in which the seller has a single good for sale and there are $n \geq 2$ potential buyers with quasi-linear preferences for the object. Bidder i assigns a value x_i to the item. Each bidder's value is unknown to the seller and to the other bidders. Their values are independent and identically distributed according to a cumulative distribution function $F : [\underline{x}, \bar{x}] \to \mathbb{R}_+$ with $-\infty < \underline{x} < \bar{x} < \infty$. The probability density function of F, f, is continuous and strictly positive everywhere on $[\underline{x}, \bar{x}]$. The problem is *regular* in the sense that the virtual valuation function

$$\Psi_F(x) = x - \frac{[1 - F(x)]}{f(x)}$$

is strictly increasing in x. The potential buyers are aware of the distribution of values and this awareness is common knowledge. Our modeling assumptions differ from the standard ones in that the seller does not know F.

The goal of the seller is to maximize expected profits. If the seller knew F, then a second price auction with reserve price r^* implicitly defined by $\Psi_F(r^*) = 0$ would be the optimal direct mechanism. However, in our model, the seller does not know F and, hence, cannot directly set an optimal reservation price. We now provide a mechanism in which the seller elicits F from the potential buyers, at an almost zero cost, and uses this information to implement a second price auction with a reservation price that is almost always equal to r^*.

The nearly optimal auction we propose consists of two interrelated mechanisms: an elicitation mechanism and a standard auction. The elicitation mechanism is, essentially, a stochastic Vickrey–Clarke–Groves (VCG) mechanism conducted between each bidder and a dummy bidder for a lottery payoff. In the auction mechanism, the item is allocated according to a standard second price auction with a stochastic reserve price. The two mechanisms are intimately related: first, the reserve price in the auction depends on the distributions obtained from the bidders in the elicitation mechanism. Second, the lottery payoffs of the elicitation mechanism depend on the bids placed by the bidders in the auction. The mechanism we offer provides strong incentives for each bidder to report truthfully both own valuation and the distribution of values if it is believed that at least one other will do so. Specifically, truthfully report of distributions and values is a strict Bayesian Nash equilibrium of the induced game.

The rules of the game are as follows. Each agent i submits a message to the seller containing two pieces of information: a nonnegative bid, b_i, and a cumulative distribution function, G_i. The seller takes these messages and, for each i, computes the largest root of $\Psi_{G_i}(x) = 0$ that we indicate by r_i. If $\Psi_{G_i}(x)$ has no root, then the seller sets $r_i = 0$. Thus, we can think of r_i as the reserve price suggested by bidder i. Then the seller draws a random vector (p, t, k). It is common knowledge that p and t are *i.i.d.* draws from the uniform distribution on $[0, 1]$ and k is a random realization from a distribution H with full support on $(-\infty, \infty)$. The realization p is used to set-up the reserve price in the auction and the numbers (t, k) are used in the elicitation mechanism. We next formalize the two related mechanisms.

Auction mechanism: the good is allocated according to a second price auction with a stochastic reserve price given by

$$r = \begin{cases} \max\{r_1, r_2, \ldots, r_n\} & \text{if } p \leq \bar{p} \\ 0 & \text{otherwise} \end{cases}$$

where $\bar{p}$ is known by all the bidders. Thus, $\bar{p}$ is the probability that the reserve price be equal to $\max\{r_1, r_2, \ldots, r_n\}$ and the probability that r be equal to zero is just $1 - \bar{p}$. Once r is defined, the item is allocated according to the usual rule. The fact that the reserve price is zero with strictly positive probability guarantees that each bidder has strict incentives to report his value even when it is very small.

Elicitation mechanism: each bidder in the auction enters into a lottery for a chance to win a prize $w > 0$. For bidder i, the lottery depends on both G_i and (t, k). It is determined as follows: let E_i be the event that bidder $i + 1$'s bid, b_{i+1}, falls in the region $(-\infty, k]$, with $n + 1 \equiv 1$. We define $L_i(t, w)$ as a lottery where bidder i wins the prize w with probability t and it wins 0 with probability $1 - t$. If bidder i submits that the distribution G_i and (t, k) are realized, then he receives the following lottery payoff

$$m_i(G_i, t, k) = \begin{cases} w\,1(E_i) & \text{if } G_i(k) \geq t \\ L_i(t, w) & \text{otherwise} \end{cases}$$

where $1(.)$ is the indicator function that takes a value of 1 if the event E_i occurs and 0 otherwise. This lottery mechanism is similar to the mechanisms proposed by Qu (2012) in the context of eliciting probability distributions from experts.

2 This mechanism is related to the elicitation procedure presented by Becker et al. (1964). Recently, Demuynck (2013) illustrated how Karni's mechanism may be used for eliciting the mean or quantiles of a random variable.

3 O'Hagan et al. (2006) survey this literature which spans several fields.

4 See also Hartline (2012) who discusses approximation in the mechanism design and surveys some of this new literature.