Economics Letters 124 (2014) 1–8

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A note on dynamic monopoly pricing under consumption externalities

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HIGHLIGHTS

- We analyze dynamic monopoly pricing under consumption externalities.
- We in particular focus on pricing under negative externalities.
- A consumer's discount factor for past sales is incorporated as a parameter.
- The optimality of introductory pricing is robust with respect to the discount factor.
- We find an oscillation as the optimal price path under negative externalities.

ARTICLE INFO

Article history:
Received 3 December 2013
Received in revised form
4 April 2014
Accepted 11 April 2014
Available online 19 April 2014

JEL classification:
D42
L12

Keywords:
Monopoly pricing
Consumption externalities
Snob effect
Congestion pricing

ABSTRACT

We analyze dynamic monopoly pricing under consumption externalities, focusing on pricing under negative externalities. We also attempt to generalize models in the previous literature, which encompass both negative and positive externalities, by incorporating a consumer's discount factor for past sales as a parameter. Analyzing our model reveals oscillation as the optimal price path in the presence of negative externalities.

1. Introduction

We analyze optimal pricing policy in a monopoly over multiple periods in the presence of consumption externalities where consumer utility is affected by past sales volume. As consumption externalities, a positive externality (the bandwagon effect) where the more a product is sold, the more consumer utility increases, is well known (see, e.g. Leibenstein, 1950). The optimal pricing path of a monopoly in the presence of positive externalities has long received attention in the literature. Examples include Bensaid and Lesne (1996), Cabral et al. (1999), and Gabszewicz and Garcia (2008). They theoretically explain the optimality of introductory pricing often observed in the dynamic pricing of some durable goods and communication network services, where prices are monotonically increasing in periods. This is contrasted with the "Coase conjecture", which states that prices for durable goods must decline over time in the absence of consumption externalities. Although the present paper considers pricing of non-durable goods, we note that the optimality of introductory pricing is not necessarily due to the durability of goods. In fact, Gabszewicz and Garcia (2008) show an introductory pricing as the optimal path on the basis of a model that assumes a new cohort of consumers enters the market at each period.

On the other hand, as also introduced in Leibenstein (1950), we often observe negative consumption externalities (the "snob effect") where scarcity (lower sales volume) of a product increases consumer utility. Examples of durable goods exhibiting this feature include fashion goods (Pesendorfer, 1995), luxury watches, and luxury cars. Among non-durable goods, which are explored in this paper, examples of this are conspicuous goods such as stays in luxury hotels, meals at luxury restaurants, and perfumes.

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http://dx.doi.org/10.1016/j.econlet.2014.04.007

We also note that congestion is another important phenomenon that induces negative consumption externalities. Thus, our study is also closely related to the literature on congestion pricing. Although some works investigate tax and toll policies toward conspicuous and congestible goods (Naor, 1969; Corneo and Jeanne, 1997), there are some papers, including ours, that tackle pricing strategy under negative consumption externalities. For example, Grilo et al. (2001) and Amaldoss and Jain (2005a) explore pricing of conspicuous goods under a static duopoly. Lee and Mason (2001) analyze price competition in a market where consumers have heterogeneous valuations of both positive and negative network externalities. In the literature on congestion pricing, Verhoef et al. (1996) study congestion pricing in a static monopoly. Friedman (1996) and Masuda and Whang (1999) develop dynamic models of congestion and well explain price and demand oscillations. Nevertheless, to the best of our knowledge, the optimal price path of a monopoly maximizing its profit over multiple periods has not been investigated in the literature. We therefore formulate a model of dynamic pricing of a monopoly based on a typical linear demand model, and attempt to conduct more comprehensive analysis in terms of the degree of consumption externalities. We also introduce a discount factor parameter into a model of consumer evaluation of past sales. For example, for some commodities such as fashionable clothes and movies, consumers might be sensitive to recent sales trends only, which implies a lower discount factor (e.g. Vettas, 2000; Kitamura, 2010). In the context of congestion, a low discount factor implies that consumers have "static expectations" (see Friedman, 1996). In contrast, for goods likely to be durable, as well as for communication network services, consumer utility is instead affected by the accumulation of sales quantities over all previous periods (e.g. Bensaid and Lesne, 1996; Gabszewicz and Garcia, 2008). In addition, as in most of the literature, including Dhebar and Oren (1985) and Cabral et al. (1999), we also consider a firm's discount factor for future revenues. In the sense that these discount factors are parameterized in our model, our model can be seen as a generalization of models employed in such previous works.

2. Model

We assume a monopoly firm that sells a non-durable good over N ($\geq$ 3) periods. Let v be a consumer's value for this good, with v uniformly distributed on [0, 1]. The firm produces and sells this good at price p_i in each period i ($i = 1, 2, \ldots, N$). For simplicity, production and setup costs are assumed to be zero. At each period i, a new cohort of consumers enters the market and the consumer indexed by $v \in [0, 1]$ has the following utility function $U_i(v)$:

$$U_1(v) = v - p_1$$

$$U_i(v) = v - k\sum_{j=1}^{i-1}\theta^{i-1-j}q_j - p_i, \quad i = 2, \ldots, N.$$

Consumer v buys the good if and only if $U_i(v)$ is nonnegative. Let q_i be the sales volume at period i. Then, due to consumption externalities, the utility at period i ($i = 2, 3, \ldots, N$) is determined according to the quantities sold in each previous period $j = 1, \ldots, i-1$. Note that no externality is generated at period 1 (Bensaid and Lesne, 1996; Gabszewicz and Garcia, 2008). This assumption of lagged externality is justified if discerning the true demand of the good generates a significant consumption externality (Amaldoss and Jain, 2005b). In such a case, at the first period, consumers do not have any information on who uses the good or how widespread its purchase is; thus, the generated externality is too restrictive, regardless of whether it is positive or negative. However, once the good is on sale, consumers can obtain this information and thus the externality is adaptively generated with a delay. Consumers may be more motivated to purchase by discovering the popularity of the good or may be less motivated by knowing of congestion or inconspicuousness. Parameter k indicates the degree of externality, where $k > 0$ implies a negative externality and $k < 0$ implies a positive externality. We define parameter θ ($0 \leq \theta < 1$) as the discount factor on past sales. We assume $|k| < 1 - \theta$ to ensure concavity of our profit function. Under this demand structure, the firm determines a pricing policy $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$ that maximizes total profit over N periods, denoted by

$$\pi = \sum_{i=1}^{N}\gamma^{i-1}p_i q_i(p_1, \ldots, p_N), \tag{1}$$

where γ ($0 < \gamma \leq 1$) is the discount factor on future profits. Note that discount factors can be different for past and future felicities, which is justified in the literature (e.g. Caplin and Leahy, 2004).

3. Analysis

First, we explicitly express profit (1) in terms of prices $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$ and then derive the first-order condition with respect to $\boldsymbol{p}$.

For the first period, let $\bar{v}$ be the marginal consumer satisfying $U_1(\bar{v}) = 0$. Then, since $\bar{v} = p_1$, the demand of the first period is $q_1 = 1 - \bar{v} = 1 - p_1$. For subsequent periods, we can obtain the demand function as follows:

$$q_i = 1 - k\sum_{j=1}^{i-1}\theta^{i-1-j}q_j - p_i.$$

By substituting the demand function for each period into (1), the firm's profit can be expressed as a function of $\boldsymbol{p}$. In what follows, we assume an interior solution for this profit-maximization problem with regard to both price ($p_i > 0$) and demand ($0 < q_i < 1$). In fact, under the assumption $|k| < 1 - \theta$, the optimal price path is guaranteed to be an interior solution (see the Appendix for this). Thus, if $\boldsymbol{p}^*$ is the interior solution, $\boldsymbol{p}^*$, must satisfy the first-order condition as shown in Box I.

Indeed, with some algebra, (2) can be transformed into the following equation with the tri-diagonal matrix:

$$B\boldsymbol{p}^* = (1-\theta)\boldsymbol{b}', \quad \text{where} \tag{3}$$

$$B = \begin{pmatrix} \alpha' & \gamma\beta & & & & \\ \beta & \alpha & \gamma\beta & & \mathbf{0} & \\ & \beta & \alpha & \gamma\beta & & \\ & & \ddots & \ddots & \ddots & \\ & \mathbf{0} & & \beta & \alpha & \gamma\beta \\ & & & & \beta & \alpha' \end{pmatrix},$$

$$\boldsymbol{b}' = \begin{pmatrix} \dfrac{1+\gamma(1-k)(k-\theta)}{1-\theta} \\ 1+\gamma(k-\theta) \\ \vdots \\ 1+\gamma(k-\theta) \\ 1 \end{pmatrix},$$

and where $\alpha \equiv 2(1 - \gamma(k-\theta)\theta)$, $\alpha' \equiv 2 - \gamma k(k-\theta)$, and $\beta \equiv k - 2\theta$. Note that the case of $k - 2\theta = 0$ is trivial and thus excluded hereafter.

By expressing the profit function in terms of sales quantity $\boldsymbol{q}$ and deriving the first-order condition with respect to $\boldsymbol{q}$, we also can prove that the optimal solution $\boldsymbol{q}^*$ is necessarily positive under $|k| < 1 - \theta$ (see the Appendix for this). In addition, with negative externality ($k > 0$), $\boldsymbol{q}^* < \mathbf{1}$ ($\mathbf{1}$ is the N-dimensional vector where all entries are 1) is also guaranteed. In contrast, with