Economics Letters 124 (2014) 108–112

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Value and quantity data in economic and technical efficiency measurement

Maria Conceição A. Silva Portela *

CEGE - Centro de Estudos em Gestão e Economia da Universidade Católica Portuguesa, Centro Regional do Porto, Rua Diogo Botelho, 1327, 4169-005 Porto, Portugal

HIGHLIGHTS

- The use of value data in efficiency models versus quantity data is addressed.
- Value data issues are analysed for equal and differing prices across DMUs.
- Guidelines are proposed regarding the type of models to be used under both cases.
- Cost efficiency models should be preferred when data are measured in value.
- Cost efficiency under value data implicitly assumes that prices are discretionary.

ARTICLE INFO

Article history:
Received 11 November 2013
Received in revised form
22 April 2014
Accepted 23 April 2014
Available online 10 May 2014

MSC:
00-01
99-00

Keywords:
Data Envelopment Analysis
Technical efficiency
Cost efficiency

ABSTRACT

This paper calls attention to the implications of using value data in efficiency measurement through Data Envelopment Analysis (DEA). The main contributions are twofold: (i) it provides a reconciliation of the previous literature on analysing issues of quantity and value data in efficiency measurement, (ii) it provides some guidelines on what to do, when these issues arise in a data set.

1. Introduction

The main motivation to this paper stems from the fact that insufficient attention has been paid for in the efficiency literature to the implications of using cost or revenue data in technical and economic efficiency assessments. Technical or productive efficiency of a firm "means its success in producing as large as possible an output from a given set of inputs" Farrell (1957, p. 254). Clearly one can also define productive efficiency as the success of firms in using as small as possible inputs to produce a given set of outputs, and this distinguishes a perspective of output expansion from a perspective of input contraction. When factor prices are available, and taken into account in the measurement of efficiency, the first perspective gives rise to revenue efficiency measurement, and the second perspective gives rise to cost efficiency measurement.

In many empirical settings input and/or output quantities and prices are not available as separate variables, and only value measures are available (Cross and Fare, 2009). A typical example found in many empirical applications is the use of capital value to measure the input capital. Since capital is in fact constituted by many sub-items, like buildings, vehicles, equipments, etc., for which real prices are not available or meaningful, it is usual to use some accounting procedure to measure the value of capital of a firm.

When value data are used in efficiency models it is questionable what sort of efficiency measure is being computed, since it cannot be a productive efficiency measure (that takes into account only quantities of inputs and outputs) and it cannot be a traditional cost or revenue efficiency measure (that considers that both prices and quantities of all inputs and outputs are available). As stated in

* Tel.: +351 226196200.
E-mail address: csilva@porto.ucp.pt.

http://dx.doi.org/10.1016/j.econlet.2014.04.023
0165-1765/

Banker et al. (2007) "since aggregate cost or revenue data reflect both quantities and prices, it is not apparent what is measured by the DEA technical efficiency model when such data are used" (p. 115). Or, put it another way, "how does the value-based DEA model relate to the quantity based DEA models? …if they do not coincide, then what exactly does the value-based DEA model measure and how do we interpret the difference?" Cross and Fare (2008).

In this paper we address situations where only value data are available (and distinguish in this case two possible situations: (i) prices are the same across production units, and (ii) prices differ across production units), and situations where for some factors there are quantity data and for others there are value data. The paper contributes to the literature in two ways: (i) by providing a reconciliation of the previous literature on analysing the use of value data in DEA models; and (ii) by providing some guidelines on what to do and not to do, when value data arise in a data set. Some of these guidelines are not new to the literature, but two of them are presented here for the first time.

2. Using value data under equal and different prices across DMUs

Consider for each Decision Making Unit (DMU) j ($j = 1, \ldots, n$) a vector $\mathbf{x_j} = (\mathbf{x_{1j}}, \ldots, \mathbf{x_{mj}})$ reflecting m inputs consumed for producing a vector of s outputs $\mathbf{y_j} = (\mathbf{y_{1j}}, \ldots, \mathbf{y_{sj}})$, where prices of inputs are given by a vector $\mathbf{w_j} = (\mathbf{w_{1j}}, \ldots, \mathbf{w_{mj}})$. A cost setting will be analysed in this paper, but the extension of the concepts to a revenue setting is straightforward. The traditional cost efficiency model for DMUo is the solution of the linear program (1), where input quantities, x_i, and the intensity variables, λ_j are the decision variables.

$$\min_{\lambda_j, x_i}\left\{\sum_{i=1}^{m} w_{io}x_i \middle| \sum_{j=1}^{n}\lambda_j x_{ij} \le x_i,\ i = 1,\ldots,m,\right.$$
$$\left.\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro},\ r = 1,\ldots,s, \lambda_j, x_i \ge 0\right\}. \quad (1)$$

From the optimal solution to (1) cost efficiency is computed as the ratio between optimal cost and observed cost (C^*/C_o). The technical efficiency for DMUo is obtained from the solution of model (2) (see e.g. Charnes et al., 1978), where constant returns to scale (CRS) are assumed.

$$\min_{\lambda_j, \theta}\left\{\theta \middle| \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{io},\ i = 1,\ldots,m,\right.$$
$$\left.\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro},\ i = r,\ldots,s,\ \lambda_j \ge 0\right\}. \quad (2)$$

The product of technical efficiency and allocative efficiency is equal to the measure of cost efficiency, and therefore allocative efficiency can be retrieved residually, after solving models (1) and (2). The above models assume that factor prices are known for each input and that these may be different across DMUs, which are considered price takers. In what follows we will assume that prices are not known, and therefore disaggregated price and quantity data are not available, but just value data on inputs are available.

2.1. Equal prices across DMUs

When prices are unknown there is the special case where prices are the same across all units (i.e. $w_{ij} = w_i$). Under this circumstance the technical efficiency model (2) is equivalent to model (3), where the quantities of each input i are replaced by the costs of each input i ($C_{ij} = w_i x_{ij}$).

$$\min_{\lambda_j, \theta}\left\{\theta \middle| \sum_{j=1}^{n}\lambda_j C_{ij} \le \theta C_{io},\ i = 1,\ldots,m,\right.$$
$$\left.\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro},\ r = 1,\ldots,s,\ \lambda_j \ge 0\right\}. \quad (3)$$

The equivalence between models (2) and (3) is easily proved, as the constraints for each input ($i = 1, \ldots, m$) $\sum_{j=1}^{n}\lambda_j C_{ij} \le \theta C_{io} \Leftrightarrow \sum_{j=1}^{n}\lambda_j w_i x_{ij} \le \theta w_i x_{io}$, which in turn equals $\sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{io}$.

Similarly model (4) is equivalent to (1), when prices for each input i are the same across DMUs, being the proof similar to the above.

$$\min_{\lambda_j, C_i}\left\{\sum_{i=1}^{m} C_i \middle| \sum_{j=1}^{n}\lambda_j C_{ij} \le C_i,\ i = 1,\ldots,m,\right.$$
$$\left.\sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro},\ r = 1,\ldots,s,\ \lambda_j, C_i \ge 0\right\}. \quad (4)$$

The above leads us to our first proposition:

Proposition 1. *When prices are equal across DMUs, cost efficiency and technical efficiency can be computed using cost data rather than quantity and price data. This means that, when prices are equal across units, in fact price information is not required to compute cost efficiency and decompose it into technical and allocative components.*

This proposition is not new to the literature. In particular Fare et al. (1990) addressed the equivalence between technical efficiency models based on quantity and value data in a profit setting, and Banker et al. (2007) addressed this equivalence in a revenue setting (see also Cross and Fare, 2008).

Not addressed in the literature, to the author's knowledge, is the equivalence between model (4) and a model where input costs are aggregated into a single input ($C_j = \sum_{i=1}^{m} C_{ij}$), where $C_{ij} > 0\ \forall i$ and j. That is, (4) is equivalent to (5) with a single aggregated input cost (where no component of total cost can be zero, as otherwise total costs are not comparable across DMUs):

$$\min_{\lambda_j, C}\left\{C \middle| \sum_{j=1}^{n}\lambda_j C_j \le C,\ \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro},\ r = 1,\ldots,s,\ \lambda_j, C \ge 0\right\}. \quad (5)$$

Proof. The optimal solution to model (5) yields variable $C > 0$, and the optimal solution to model (4) yields $C_i > 0,\ \forall i$ (otherwise it would be possible to produce positive output with some zero costs). Consider now the dual of model (4) in (6).

$$\max_{v_i, u_r}\left\{\sum_{r=1}^{s} u_r y_{ro} \middle| -\sum_{i=1}^{m} v_i C_{ij} + \sum_{r=1}^{s} u_r y_{rj} \le 0,\right.$$
$$\left. j = 1,\ldots,n,\ v_i \le 1,\ i = 1,\ldots,m\right\}. \quad (6)$$

Through complementary slackness conditions of linear programming the constraints $v_i \le 1$ corresponding to the basic variables C_i are binding. Similarly the dual of model (5) is shown in (7), where through complementary slackness conditions of linear programming the constraint $v \le 1$ corresponding to the basic variable C is binding.

$$\max_{v, u_r}\left\{\sum_{r=1}^{s} u_r y_{ro} \middle| -vC_j + \sum_{r=1}^{s} u_r y_{rj} \le 0, j = 1,\ldots,n,\ v \le 1\right\}. \quad (7)$$