Economics Letters 120 (2013) 592–595

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Semiparametric selection of seasonal cointegrating ranks using information criteria

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HIGHLIGHTS

- I apply the semiparametric cointegrating rank selection to seasonal cointegration.
- I investigate some properties of the information criteria under the proposed model.
- The proposed procedure is convenient for practical implementation in empirical work.
- Especially, the BIC and HQ criteria show superior performances in most cases.

ARTICLE INFO

Article history:
Received 17 April 2013
Received in revised form
3 June 2013
Accepted 23 June 2013
Available online 28 June 2013

JEL classification:
C12
C22
C32

Keywords:
Seasonal cointegration
Seasonal error correction model
Seasonal unit roots

ABSTRACT

We consider the use of information criteria (IC) on the basis of a semiparametric seasonal error correction model for selecting seasonal cointegrating ranks. Some limit properties of the IC are considered and, through a small Monte Carlo simulation, we evaluate the performance of the IC.

1. Introduction

Econometricians have recently shed new light on the use of information criteria (IC) instead of traditional likelihood ratio (LR) tests for identifying cointegrating rank (CIR). They find that unlike the LR tests, IC may not require complete model specification. The specification includes a lag order of vector autoregression and an appropriate significance level to implement a test procedure. Conveniently, the use of IC allows a nonparametric or semiparametric short memory component and a reduced rank regression with only a single error correction (EC) term (Cheng and Phillips, 2009).

Kapetanios (2004) derived an asymptotic distribution of the CIR estimate by using the Akaike information criterion (AIC). Wang and Bessler (2005) compared the performance of CIR determination by IC such as the AIC and the Schwarz Bayesian information criterion (BIC), with results determined by the LR tests. Cheng and Phillips (2009) provided the general limit properties of IC for semiparametric CIR selection. However, all these studies are restricted to nonseasonal cointegration and not extended to seasonal cointegration.

One commonly-used CIR test in seasonal cointegration is the LR test (see, for example, Seong, 2013, and the references cited therein). This test still requires a predetermined lag order of vector autoregression because of which practitioners in empirical work are confronted with difficulties in procedures related to seasonal cointegration.

In this article, we consider the use of IC based on a semiparametric model for selecting seasonal CIR, investigate IC limit properties, and conduct Monte Carlo simulations to evaluate the IC performance.

2. The LR test for seasonal CIR

We consider a vector autoregressive (VAR) model of order p, VAR(p), for an n-dimensional process X_t satisfying

$$\Pi(L)X_t = \left(I_n - \sum_{j=1}^{p} \Pi_j L^j\right) X_t = \varepsilon_t, \quad t = 1, \ldots, T \tag{1}$$

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0165-1765/$ – see front matter
http://dx.doi.org/10.1016/j.econlet.2013.06.031

where ε_t are independent and identically distributed (IID) with $E(\varepsilon_t) = 0$ and $\text{cov}(\varepsilon_t) = \Omega$; I_n denotes an $n \times n$ identity matrix; $\Pi(L)$ is a polynomial matrix and L is a lag operator such that $L^j X_t = X_{t-j}$. We assume that the roots of determinant $|\Pi(z)| = 0$ are on or outside the unit circle, and X_t are observed on a quarterly basis with no deterministic terms.[1]

We know from Cubadda (2001) that if series are seasonally cointegrated of order (1, 1) at frequencies 0 ($z = 1$), π ($z = -1$), $\pi/2$ ($z = i$), and $3\pi/2$ ($z = -i$) where $i = \sqrt{-1}$, then model (1) can be rewritten in the error correction model (ECM)

$$\Gamma(L)X_t^{(0)} = \alpha_1\beta_1^* X_{t-1}^{(1)} + \alpha_2\beta_2^* X_{t-1}^{(2)} + \alpha_3\beta_3^* X_{t-1}^{(3)} + \varepsilon_t \quad (2)$$

where

$$X_t^{(0)} = (1-L)(1+L)(1+iL)X_t,$$
$$X_t^{(1)} = (1+L)(1+iL)X_t/\{2(1+i)\}$$
$$X_t^{(2)} = (1-L)(1+iL)X_t/\{2(1-i)\},$$
$$X_t^{(3)} = (1-L^2)X_t/(2i)$$

α_j and β_j are complex-valued $n \times r_j$ matrices with rank r_j for $j = 1, 2, 3$, and $0 \le r_j \le n$; β_j^* denotes the conjugate transpose of β_j; and $\Gamma(L)$ is a $(p-3)$ order matrix polynomial.

The seasonal CIR at unit root $z_m = \exp(i\theta_m)$ with frequency $\theta_m \in [0, \pi]$ is the number of linearly independent cointegrating relations among the components of X_t, and the same as the rank r_m of $\Pi(z_m) = -\alpha_m\beta_m^*$. The hypothesis for identifying seasonal CIR is as follows:

$$H_0: r_m \le r_m^0 \quad \text{versus} \quad H_1: r_m > r_m^0 \quad \text{for some } 0 \le r_m^0 < n.$$

To test the hypothesis, the LR test statistic is obtained by

$$LR(r_m^0) = -T \log \left(\frac{\max_{H_1} |\tilde{\Omega}|}{\max_{H_0} |\tilde{\Omega}|} \right)$$

where $\tilde{\Omega}$ denotes the estimator of the covariance matrix Ω of ε_t in model (1) with a known or pre-specified order p under the hypothesis H_0 or H_1. The test is sequentially performed, starting from $r_m^0 = 0$, and its asymptotic distribution and critical value at a given significance level are sought.

3. The semiparametric seasonal ECM and IC

The LR test in Section 2 for identifying seasonal CIR follows a parametric or probabilistic procedure. This is likely to have problems of low power and size distortions when, for example, the lag order of VAR is misspecified or errors are not IID (Wang and Bessler, 2005).

An alternative to the traditional test procedure is to consider various IC under a semiparametric approach. Such an application of the model selection approach has been investigated by a number of studies in the literature, although only in the context of nonseasonal cointegration. Among others, Cheng and Phillips (2009) proposed a semiparametric ECM that allows the error term to be weakly stationary and investigated asymptotic properties of IC under this model. In this paper, we apply the concept to seasonal cointegration.

In order to construct a semiparametric ECM for seasonal cointegration, we retain four terms of the ECM in (2), including the three EC terms and one lagged term of $X_t^{(0)}$, and incorporate the other terms into the error term. In case of monthly data, seven EC terms and five lagged terms of $X_t^{(0)}$ are retained. Note that the retention of lagged terms results from the fact that the ECM in (2) uses only the roots on the upper half unit circle. We then obtain the semiparametric seasonal ECM

$$X_t^{(0)} = \alpha_1\beta_1^* X_{t-1}^{(1)} + \alpha_2\beta_2^* X_{t-1}^{(2)} + \alpha_3\beta_3^* X_{t-1}^{(3)} + \Gamma_1 X_{t-1}^{(0)} + \xi_t \quad (3)$$

where ξ_t is a weakly stationary time series with zero mean. We treat the ECM in (3) semiparametrically with regard to ξ_t and estimate the seasonal CIRs r_1, r_2, and r_3 directly in the ECM by IC. Specifically, we select ranks as follows: the ECM in (3) is estimated for all values of $r_m = 0, 1, \ldots, n$ just as if ξ_t were a martingale difference, and r_m is chosen to minimize the IC as if the ECM were a correctly specified parametric framework up to the rank parameter r_m.

Let $\hat{\Omega}(r_m)$ be the residual covariance matrix from the estimation of ECM in (3)

$$\hat{\Omega}(r_m) = T^{-1} \sum_{t=1}^{T} (R_t^{(0)} - \hat{\alpha}_m\hat{\beta}_m^* R_t^{(m)})(R_t^{(0)} - \hat{\alpha}_m\hat{\beta}_m^* R_t^{(m)})'$$
for each $r_m = 0, 1, \ldots, n$

where $R_t^{(0)}$ and $R_t^{(m)}$ denote the residuals from regressing $X_t^{(0)}$ and $X_{t-1}^{(m)}$ on the other regressors from the ECM in (3). The criterion used to evaluate seasonal CIR takes the simple form:

$$IC(r_m) = \log\left|\hat{\Omega}(r_m)\right| + C_T T^{-1}(2nr_m - r_m(r_m+1)/2) \quad (4)$$

where coefficient $C_T = \log(T)$, $2\log\log T$, or 2 corresponds to the BIC, HQ (Hannan and Quinn, 1979), and AIC penalties, respectively. The degrees of freedom term $2nr_m - r_m(r_m+1)/2$ accounts for the $2nr_m$ elements of the matrices α_m and β_m that have to be estimated, adjusted for the normalization restriction on β_m, $\hat{\beta}_m^* S_{m,m}\hat{\beta}_m = I_{r_m}$ where $S_{m,m} = \sum_{t=1}^{T} R_t^{(m)} R_t^{(m)*}$. Therefore, model evaluation based on the IC then leads to the seasonal CIR selection criteria

$$\hat{r}_m = \arg\min_{0 \le r_m \le n} IC(r_m) \quad \text{for } m = 1, 2, 3.$$

The following proposition states the sufficient conditions that the IC are weakly consistent for selecting seasonal CIR.

Proposition 1. *The IC are weakly consistent if the penalty term in (4) satisfies* (i) $C_T \to \infty$ *and* (ii) $C_T = o(T)$ *as* $T \to \infty$. *Conditions* (i) *and* (ii) *are needed to prevent the seasonal CIR from being overestimated and underestimated, respectively.*

Similar to nonseasonal cointegration, the proposition shows that BIC, HQ, and other IC with $C_T \to \infty$ and $C_T = o(T)$ are all consistent for the selection of seasonal CIR without having to specify a full parametric model such as the lag order p. The AIC penalty is fixed at $C_T = 2$ for all T, and then it is inconsistent by selecting models with excessive seasonal CIR with positive probability as $T \to \infty$. However, the AIC asymptotically never underestimates seasonal CIR.

4. Monte Carlo simulations

We conduct Monte Carlo simulations to evaluate the finite sample performance of the three standard IC (BIC, AIC, and HQ) under various generating mechanisms for the short memory component ξ_t and compare the performance with that of the LR test at a 5% significance level.

The first data-generating process (DGP I) is, as VAR(6) for a two-dimensional process,

$$Y_t = \alpha_1\beta_1' U_{t-1} + \alpha_2\beta_2' V_{t-1} + \alpha_3\beta_2' W_{t-1} + \alpha_4\beta_2' W_{t-2} + \xi_t,$$
$$\xi_t = \Psi_1 Y_{t-1} + \Psi_2 Y_{t-2} + u_t$$

[1] Models with other seasonal periods (e.g., monthly) and models with deterministic terms, that may contain a constant, a linear trend, or seasonal dummies, can easily be implemented, as in Cubadda (2001) and Seong et al. (2006).