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A note on input congestion

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HIGHLIGHTS

- The notion of effective space is introduced and related to input congestion.
- Congestion may occur at the industry level when it is absent at the firm level.
- Profit maximization rules out congestion for firms, but not for the industry.
- Important implications for aggregate efficiency and for regulating externalities.

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ABSTRACT

The notion of effective space is introduced, and input congestion is explained by economic activities' exhaustion of effective space. In this setting, I show that profit maximization is inconsistent with input congestion at the *firm level*, but not necessarily with input congestion at the *industry level*, when effective space is shared among producers.

1. Introduction

Input congestion is present when there are negative returns to inputs in production, i.e., when employment of additional units of inputs obstructs the output. The concept originates from Färe and Svensson (1980), who related it to the law of diminishing returns by Turgot. In the classical treatment of this law, space (land) is a fixed factor on which variable factors cause congestion (overcrowding). In line with the law, Färe and Svensson (1980) evaluated input congestion when some inputs are fixed while others are variable.

Input congestion has become an important topic in the Data Envelopment Analysis (DEA) literature. Unlike Färe and Svensson (1980), the DEA literature has not emphasized the role of fixed inputs in congestion. See Cherchye et al. (2001) for a critical discussion on congestion in DEA.

The current paper explains input congestion in a way similar to the law of diminishing returns, but instead of considering space as an essential input it considers *effective space* as an essential input. The concept of effective space concerns the *quality* of the physical space which, in contrast to the physical space itself, diminishes as a result of human activities. Real-life examples are numerous: plowing contributes to degradation of land for cultivation; the quality of grazing land is negatively related to the number of animals; the quality of a road, both in terms of decay and of average speed, depends upon traffic; feed spills from aquaculture reduce the water quality and contribute to fish diseases.

Production analysis is generally concerned with inputs which the entrepreneur exercises effective control over (Chambers, 1988). Effective space may not conform to this requirement and is therefore usually not accounted for. In the following, I consider "congestion models" (e.g., in the DEA literature) that do not incorporate effective space to be reduced forms (Murty et al., 2012) of the "true technology" that incorporates effective space. In this setting, I show that a production function exhibiting *free disposability* of inputs allows detecting input congestion when increases in economic activity come at the expense of effective space. I find that

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profit maximization is inconsistent with input congestion at the *firm level*, but not necessarily with congestion at the *industry level*, when effective space is shared among producers.

2. Congestion measurement by the reduced form technology

Consider an industry consisting of $k = (1, 2)$ firms. Each firm is represented by a (reduced-form) production function $\tilde{f}^k(x^k)$ that converts an input $x^k \in \Re_+$ into an output $y^k \in \Re_+$. I assume that $\tilde{f}^k$ is differentiable and introduce the axiom of free disposability of inputs:

$$\partial y^k/\partial x^k = \partial \tilde{f}^k(x^k)/\partial x^k \geq 0, \quad k = (1, 2). \tag{1}$$

Eq. (1) rules out input congestion as the output is assumed not to decrease in the input. Congestion, on the other hand, arises when the marginal product is negative. Formally, for firm k,

$$\begin{aligned} &\partial y^k/\partial x^k = \partial \tilde{f}^k(x^k)/\partial x^k \geq 0 \quad \text{if } x^k \leq \mu^k \\ &\partial y^k/\partial x^k = \partial \tilde{f}^k(x^k)/\partial x^k < 0 \quad \text{if } x^k > \mu^k. \end{aligned} \tag{2}$$

Notice that input congestion is considered at the firm level. Eq. (2) resembles Färe and Svensson's (1980) concept of monotone output-limitational (MOL) congestion, which states that the technology is congested if it fails to satisfy the free disposability assumption. It is also related to Briec and Kerstens' (2006) S-disposability axiom, which treats free disposability of inputs as a local technology property.

3. A new look at input congestion

Recall that Eq. (2) defines input congestion without relating it to fixed inputs. Hence, it is in line with current treatments on input congestion in DEA. In the following, I aim at assessing the underlying determinants of Eq. (2) by establishing a more comprehensive model to study the dynamics of input congestion.

Denote the physical space by $b \in \Re_+$ and effective space by b^k, where $b^1 = b^2$ (i.e., effective space is the same for both producers). Effective space is a function of the *quantity* of space available and the firms' employment of the marketable input, $x^1 + x^2$. The comprehensive technology for producer k is defined by

$$\begin{aligned} y^k &= f^k(x^k, b^k) \\ b^k &= g(x^1 + x^2, b) \end{aligned} \Leftrightarrow y^k = f^k(x^k, g(x^1 + x^2, b)), \quad k = (1, 2). \tag{3}$$

I assume that f^k is everywhere twice-continuously differentiable; finite, non-negative, real valued, and single valued for all non-negative and finite input vectors; and zero when the input vector is the zero vector. The two functions f^k and g are assumed to satisfy axioms (i)–(v).

(i) Free disposability of inputs: $\partial f^k/\partial x^k \geq 0;\ \partial f^k/\partial b^k \geq 0,\ k = (1, 2)$.
(ii) Concavity: $\partial^2 f^k/\partial x^{k^2} \leq 0;\ \partial^2 f^k/\partial b^{k^2} \leq 0,\ k = (1, 2)$.
(iii) Quality degradations: $\partial g/\partial x^1 = \partial g/\partial x^2 < 0$.
(iv) Linearity: $\partial^2 g/\partial x^{1^2} = \partial^2 g/\partial x^{2^2} = 0$.
(v) Quality increases in space: $\partial g/\partial b \geq 0$.

The two first axioms are standard in production theory. Axioms (iii)–(v) imply that the quality of space degrades linearly[1] with economic activity and increases in (unspoiled) space. In addition, I assume that effective space is an abundant factor when only one of the two producers operates (and maximizes profits). Let x^{l^*} be the profit-maximizing input vector for one of the two firms, and define the following.

(vi) Abundance: $\lim_{x^k \to 0} \frac{\partial f^k(x^k, g(x^k + x^{l^*}, b))}{\partial g} = 0,\ l \neq k,\ l, k = (1, 2)$.

I derive the marginal product of x^k by taking the first-order derivative of Eq. (3):

$$\frac{\partial y^k}{\partial x^k} = \underbrace{\frac{\partial f^k}{\partial x^k}}_{\geq 0} + \underbrace{\frac{\partial f^k}{\partial g}\frac{\partial g}{\partial x^k}}_{\leq 0} \gtreqless 0, \quad k = (1, 2). \tag{4}$$

The first term (the *direct* effect) in Eq. (4) represents the marginal productivity of x^k in the production of y^k, which is positive by axiom (i). The second term (the *indirect* effect) represents reductions in y^k due to the exhaustion of effective space. It is the product of $\partial f^k/\partial g$, the marginal productivity of effective space in the production of y^k, and $\partial g/\partial x^k$, the exhaustion of effective space by a marginal increase in economic activity. According to Eq. (4), input congestion occurs when the indirect effect *dominates* the direct effect. Note that axiom (vi) is sufficient (but not necessary) to ensure that there is no congestion when x^k approaches zero, since the indirect effect is zero by axiom (vi) whereas $\partial f^k/\partial x^k$ is greater than or equal to zero by axiom (i). By relating Eq. (4) to Eq. (2), congestion in the reduced-form technology from Section 2 can be explained in terms of exhaustion of effective space.

Next, I derive the second-order derivative of Eq. (3) by taking the derivative of Eq. (4) and applying axioms (i)–(iv):

$$\frac{\partial^2 y^k}{\partial x^{k^2}} = \underbrace{\frac{\partial^2 f^k}{\partial x^{k^2}}}_{\leq 0} + \underbrace{\frac{\partial^2 f^k}{\partial g^2}\left(\frac{\partial g}{\partial x^k}\right)^2}_{\leq 0}, \quad k = (1, 2). \tag{5}$$

Eqs. (4) and (5) imply that the *direct effect* is increasing concave, while the *indirect effect* is decreasing concave. These curvature properties ensure that the point of congestion – if any – is global, since the indirect effect dominates the direct effect from this point on. However, they are not sufficient for ensuring that congestion takes place as x^k approaches infinity. The reason is that effective space becomes exhausted (zero) when x^k is sufficiently large. If the direct effect dominates the indirect effect at this point, there is no congestion. Congestion may, however, be ensured by imposing the Inada (1963) condition that the marginal productivity of effective space approaches infinity as effective space approaches zero. Alternatively, effective space can be treated as an essential input; see Shephard (1970). The latter approach will imply a severe form of congestion, namely that no production can take place when effective space is exhausted. This is similar to output-prohibitive (OP) congestion in the terminology of Färe and Svensson (1980).

Assume now that the two producers are profit maximizers and face the same prices, where $p \in \Re_+$ denotes the output price and $w \in \Re_+$ denotes the input price. If the (quantitative) space is (quasi)fixed, the profit-maximization problem for producer k is

$$\pi^k(p, w, x^l, b) = \max_{x^k} pf^k(x^k, g(x^k + x^l, b)) - wx^k, \quad l \neq k,\ l, k = (1, 2), \tag{6}$$

with first-order condition

$$\frac{\partial \pi^k}{\partial x^k} = p\left(\frac{\partial f^k}{\partial x^k} + \frac{\partial f^k}{\partial g}\frac{\partial g}{\partial x^k}\right) = w, \quad k = (1, 2). \tag{7}$$

The first-order condition states that the value of the marginal productivity of x^k equals the factor price in optimum. Since the two prices are non-negative, Eq. (7) cannot hold with equality when the

[1] Linearity is assumed for convenience, as it simplifies the expositions. The results in the paper may also be derived under convexity, i.e., under decreasing exhaustion of effective space.