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Credit limits and bankruptcy

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HIGHLIGHTS

- Consumers borrow against credit limits at a pre-approved interest rate.
- With one-period debt, these credit limits and default are jointly determined.
- Endogenous limits and positive default coexist.
- The outcome requires: pooling, fixed intermediation costs, and private information.
- Numerically illustrates factors affecting credit limits and bankruptcy.

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ABSTRACT

Credit cards offer a limit, rather than a specific loan size, at a pre-approved interest rate. This paper studies the determination of these credit limits jointly with default in the presence of one-period debt. I adapt the standard incomplete markets macroeconomic model of one-period unsecured debt with the optimal choice of credit limit. Endogenous limits and positive default coexist. A numerical exercise illustrates the consequences of various factors for indebtedness, credit limits, and bankruptcy.

1. Introduction

Borrowing limits, for loans of varying size at a pre-approved interest rates, are one defining feature of credit-card unsecured consumer credit. The occurrence of default is another characteristic of these loans. However the existing macroeconomics literature with standard one-period debt contracts does not account for these two features jointly. This paper shows conditions such that positive default coexists with endogenous borrowing constraints.

This paper adapts the workhorse model of idiosyncratic risk with incomplete markets à la Aiyagari (1994) with consumer default and an endogenous borrowing limit. With a loan manager choosing the limit, the required conditions are that banks must buy into pools of loans, some intermediation costs become fixed when the limit is set, and households' characteristics are private information. A numerical example illustrates an application.

This paper contributes to a quantitative literature on consumer bankruptcy with one-period debt. The early papers also have pooling but the credit limit is exogenous.[1] Other works endogenise the credit limit but bankruptcy is zero.[2] Chatterjee et al. (2007) depart from pooling and assume instead markets for each loan size so there are not the credit limits with pre-approved terms I am considering.[3]

2. The model

This is an infinite-horizon production economy with incomplete markets and default risk. The household and production sides

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1 Athreya (2002) and Li and Sarte (2006).

2 For example Zhang (1997), Mateos-Planas and Seccia (2006) or Ábrahám and Cárceles-Poveda (2010).

3 Similarly Livshits et al. (2007) and Athreya et al. (2009), or Athreya et al. (2012) on private information.

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of the economy are standard. The new features stem from the intermediation sector.

2.1. Households and firms

There is a continuum unit-mass of individual households, each with one unit endowment of time per period which can be divided between leisure l and working in the market $1-l$. Labour productivity in the market, s, can take on two values s^1 and s^2 with $s^1 > s^2$, and follows a Markov process with transition probabilities $\pi^s(s' \mid s)$ for $s', s \in \{s^1, s^2\}$. The wage per efficient unit is w. An individual with a clean bankruptcy record receives a non-discretionary liability shock $x \in \{x^1, x^2\}$, where $x^1 < x^2$, with probability $\pi^x(x)$.

A household can trade one-period contracts depending on her bankruptcy state, $z \in \{0, 1\}$, and her bankruptcy decision, $d \in \{0, 1\}$. With $z = 0$ and $d = 0$ she has a clean bankruptcy record and can save and borrow at the market interest rates. The interest rate on savings is r. Borrowers use a credit line which specifies the interest spread λ and the borrowing limit as a lower bound on asset positions $\underline{b} \le 0$. If $z = 1$, the bankruptcy flag prevents her from borrowing. If $z = 0$ and $d = 1$, she files for bankruptcy and is unable to either borrow or lend. A clean agent, i.e. $z = 0$, may decide either not to repay her negative bond balances and non-discretionary expenses by making the default (or bankruptcy) choice $d = 1$ or, otherwise, $d = 0$. In the former case, $z' = 1$ in the next period; later on, $z' = 0$ with probability ρ.

Preferences are defined over consumption, c, and leisure l, with period utility $u(c, l) = \frac{1}{1-\sigma}\left(c^{\eta} l^{1-\eta}\right)^{1-\sigma}$ where $\eta \in [0, 1]$ and $\sigma > 0$. Defaulting or being bankrupt carries a disutility $c_z > 0$. Lifetime utility is the expected sum of period utilities over an infinite horizon discounted at rate $\beta \in (0, 1)$.

Aggregate output is produced competitively via labour N and capital K into a Cobb–Douglas production function, with α the capital share. This output can be consumed, purchased for non-discretionary expenses and banks' costs, and invested in capital. The rate of depreciation of capital is δ.

2.2. Intermediation

Credit conditions are determined in two stages. First, free entry competition in banking determines the interest rate of the credit pools. Second, taking as given the interest rate, the loan manager independently chooses the credit limit that maximises the bank's value.

For the bank/investor, the value of non-performing loans is a proportion θ which denotes the average default rate. There are intermediation costs which, for exposition purposes, are of two kinds. One subtracts a proportion κ_V from the gross return $1+r-\lambda$; another subtracts a fraction κ_F from the recovery rate $1-\theta$. So the bank's net cash-flow per unit invested/lent is

$$CF_{inv}(\lambda, \theta) \equiv (1 + r + \lambda - \kappa_V)(1 - \theta - \kappa_F) - (1 + r). \quad (1)$$

The loan manager deals with credit lines to a representative cross-section of households. It takes as given the interest rate but can adjust the credit limit b^*. The amounts lent and defaulted can be written, abstractly, as $L(b^*; \lambda, \underline{b})$ and $L^d(b^*; \lambda, \underline{b})$ respectively, functions of the particular limit chosen b^*, and also aggregate lending spread λ and prevailing credit limit $\underline{b}$. We assume the cost component κ_F is fixed at this stage, so the manager's relevant cash-flow becomes

$$CF_{man}(b^*; \lambda, \underline{b}) \equiv (\lambda - \kappa_V)L(b^*; \lambda, \underline{b}) - (1 + r + \lambda - \kappa_V)L^d(b^*; \lambda, \underline{b}). \quad (2)$$

Equilibrium in intermediation determines θ, λ and $\underline{b}$ as follows. Free entry implies the zero-profit condition

$$CF_{inv}(\lambda, \theta) = 0. \quad (3)$$

The proportion of non-performing loans determines the default probability

$$\theta = \frac{L^d(\underline{b}; \lambda, \underline{b})}{L(\underline{b}; \lambda, \underline{b})}. \quad (4)$$

The representative loan manager's optimal decision solves the fixed-point problem

$$\underline{b} = \arg\max_{b^*} CF_{man}(b^*; \lambda, \underline{b}). \quad (5)$$

Pooling and private information are necessary assumptions. Removing either would result in different interest rates serving each level of debt as in Chatterjee et al. (2007), thus rendering immaterial the idea of credit limit.[4] The fixed cost κ_F is needed since it makes the loan manager tolerant of above-average default at the borrowing limit.

3. General equilibrium

An equilibrium can be found in two steps, first, as standard for a given credit limit, including conditions (3) with (4),[5] and, second, given the equilibrium conditions, by verifying that profitable deviations away from that debt limit are ruled out in the sense of (5).

3.1. Equilibrium for given credit limit

Consider one credit limit $\underline{b}$. A stationary equilibrium can be formulated recursively. The individual state space is $S \equiv \mathbb{R} \times \{s_1, s_2\} \times \{x_1, x_2\} \times \{0, 1\}$ with elements $a \equiv (b, s, x, z) \in S$ and $\mathcal{A}_S$ its Borel σ-algebra. An equilibrium is a probability measure Φ on the measurable space $(S, \mathcal{A}_S)$, a deposit interest rate r, a wage rate w, a lending spread λ, the default risk rate θ, a value function $v(a)$, decision rules for bonds $b'(a)$, leisure $l(a)$, and defaulting $d(a)$, and face and defaulted value of loans, $L(\underline{b}; \lambda, \underline{b})$ and $L^d(\underline{b}; \lambda, \underline{b})$, such that: (i) Given r, λ, w and $\underline{b}$, the functions $b'(\cdot)$, $l(\cdot)$, $d(\cdot)$ and $v(\cdot)$ solve the household's problem; (ii) Given w and r, firms' choice of K/N equalises marginal product and input prices, $w = (1-\alpha)(K/N)^{\alpha}$ and $r = \alpha(K/N)^{\alpha-1} - \delta$; (iii) Markets for assets and labour clear $\int_S b \mathrm{d}\Phi = K$ and $\int_S (1 - l(b, s, x, z)) s \mathrm{d}\Phi = N$; (iv) Stationary distribution: $\Phi(A) = \int_S Q(a, A)\mathrm{d}\Phi$ for $A \in \mathcal{A}_S$, with $Q : S \times \mathcal{A}_S \to [0, 1]$ the transition function reflecting decisions rules and transition probabilities; (v) Banking zero profits: λ and θ satisfy the zero-profit condition (3), with (1); (vi) Aggregate default risk θ satisfies (4), with $L(\underline{b}; \lambda, \underline{b})$ and $L^d(\underline{b}; \lambda, \underline{b})$ derived from the distribution Φ and household decision rules.

The default rule $d(b, s, x, 0)$ will imply that bankruptcy occurs if and only if debt $(-b)$ is above a certain threshold denoted $b(s, x)$, and lower income implies higher risk. As for the face and defaulted values of loans, L and L^d in (4), denote by $\pi^d(b', s, x)$ tomorrow's default probability to type (s, x) borrowing b', denote by $h(b', s, x)$ the density of type (s, x) who will borrow b', for $b' \in [\underline{b}, 0]$. Let $H(\underline{b}, s, x)$ denote the mass of type (s, x) who choose the limit $\underline{b}$. Consistency with the equilibrium Φ, $\underline{b}$, and $d(\cdot, \cdot, \cdot, \cdot)$ and $b'(\cdot, \cdot, \cdot, \cdot)$ means

$$\begin{aligned}
\pi^d(b', s, x) &\equiv \sum_{s', x'} d(b', s', x', 0)\pi^x(x')\pi^s(s' \mid s) \\
h(b', s, x) &\equiv \phi(b'^{-1}(b', s, x, 0), s, x, 0) \quad (6)\\
H(\underline{b}, s, x) &\equiv \Phi((\tilde{b}, \tilde{s}, \tilde{x}, \tilde{z}) \in S : b'(\tilde{b}, \tilde{s}, \tilde{x}, \tilde{z}) = \underline{b}, \\
&\qquad \tilde{s} = s, \tilde{x} = x, \tilde{z} = 0)
\end{aligned}$$

where $\phi(\cdot)$ is the density associated with Φ, and $b'^{-1}(\cdot)$ the inverse of the policy function. The values relevant for evaluating (4)

[4] But fully private information is stronger than required.

[5] Like in Athreya (2002) and Li and Sarte (2006).