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# Implementation lag and the investment decision

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## HIGHLIGHTS

- We model investment under uncertainty with implementation lag.
- We examine the effect of the lag on the optimal investment trigger.
- The conventional result is that the lag raises investment trigger.
- Sufficient project reversibility or high growth rate can overturn the conventional result.
- The effect of uncertainty on investment with implementation lag is similar.

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ABSTRACT

The effect of implementation lag on investment trigger depends on project reversibility and growth rate. Conventional results (that longer lag and greater uncertainty raise investment trigger) are overturned if the project is sufficiently reversible and/or has a high enough growth rate.



## 1. Introduction

When a company invests in a project, there is generally an implementation lag before cash flows begin. Two papers examine rigorously the effect of implementation lag on corporate investment. In Bar-Ilan and Strange (1996), an existing firm can suspend and restart operations, with a lag between the restart decision and the start of the cash flow stream. They show that (i) increased uncertainty can lower the restart trigger, and (ii) as the lag is increased, the restart trigger generally falls (except for very long lags). On the other hand, Alvarez and Keppo (2002), who examine the initial investment decision (under implementation lag) with no abandonment option, show that greater uncertainty and longer lag both result in delayed investment (higher investment trigger).

The objective of this paper is to reconcile these two models, by examining a firm making a one-time investment decision with an option to abandon the project. Implementation lag is important in new projects, which require large constructions and other time-consuming procedures, but less important for suspension and restart decisions because no significant construction is involved. Therefore, our model examines the initial investment, unlike Bar-Ilan and Strange (1996).[1] Also, most firms have the option to abandon the project; in fact, inability to exit is inconsistent with limited liability. Therefore, the firm in our model has the option to abandon the project, unlike Alvarez and Keppo (2002). Thus, our model contains realistic features of both the models above.

There is some research on the role of implementation lag or "time-to-build" in the uncertainty–investment relationship. Aguerrevere (2003) shows that, for incremental investment with time-to-build, higher uncertainty may encourage firms to increase

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[1] Another difference is that Bar-Ilan and Strange (1996) assume that the investment cost is incurred not when the investment is made but when the cash flows begin (i.e. after the lag), whereas we assume that the cost is incurred when the investment is made. Although the reality is somewhere between the two extremes, the latter seems more reasonable (and is consistent with Aguerrevere, 2003).

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capacity. On the other hand, Majd and Pindyck (1987) show, in a model of sequential investment, that time-to-build magnifies the negative effect of uncertainty on investment.

We show that the conventional results (i.e., both longer implementation lag and greater uncertainty raise investment trigger) can be overturned if the project is sufficiently "reversible" and/or has a high enough growth rate.

## 2. The model

The firm has the option to invest in a project, which will produce one unit of output per unit time in perpetuity, or until the firm decides to permanently shut down operations. The output price $p_t$ follows a lognormal process:

$$dp/p = \mu dt + \sigma dz \tag{1}$$

where $\mu$ and $\sigma$ are the drift and volatility of the price process, respectively, and $dz$ is the increment of a standard Wiener process. As in Bar-Ilan and Strange (1996), future cash flows are all discounted at a rate of $r$, where $r > \mu$ (for convergence).

The operating cost is $\$w$ per unit; hence, the net cash flow from the project is $(p - w)$ per unit time. When the firm invests in the project, it has to pay $\$I$. The firm will exit the business when $p$ falls sufficiently (say, to $p_e$); thus, $p_e$ is the exit trigger and is determined by the company. When the firm exits, it gets back the salvage value $\$zI$, where $0 \le z \le 1$. A larger salvage value $z$ makes it easier to exit; hence, it makes the project more reversible.

### 2.1. Project valuation

The (post-investment) project value is given by[2]

$$U(p) = p/(r - \mu) - w/r + Ap^{\gamma_2} \tag{2}$$

where $A$ is a constant to be determined by boundary conditions, and $\gamma_2(< 0)$ is given by

$$\gamma_2 = 0.5 - \mu/\sigma^2 - \sqrt{2r/\sigma^2 + \left(0.5 - \mu/\sigma^2\right)^2}. \tag{3}$$

In Eq. (2), $\{p/(r - \mu) - w/r\}$ is the project value with no exit option, and $Ap^{\gamma_2}$ is the value of the option to exit. Therefore, $A > 0$. To solve for $A$ and $p_e$, we use the standard boundary conditions. At the boundary $p = p_e$, the two boundary conditions are

$$\text{Value-matching}: U(p_e) = zI \qquad \text{Smooth-pasting}: U'(p_e) = 0,$$

which give

$$p_e = (w + rzI)(1 - \mu/r)/(1 - 1/\gamma_2) \tag{4}$$

$$A = \frac{-(p_e)^{1-\gamma_2}}{\gamma_2(r - \mu)}. \tag{5}$$

This completes the project valuation.

### 2.2. The investment decision

Suppose the project's cash flows start at time $\theta$ after the investment is made, i.e., the implementation lag is $\theta$. The actual payoff from the investment will then depend on the (uncertain) output price $p$ at that time, say $p(\theta)$. If, at that time, $p(\theta) > p_e$, the project will proceed, with value $U(p(\theta))$. If $p(\theta) \le p_e$, the company will abandon the project and receive the salvage value $zI$.

[2] All derivations are available on request from the authors.

Then the project value at the time of investment will depend on both $p$ and $\theta$, say $V(p, \theta)$, and is given by

$$V(p, \theta) = e^{-r\theta} \int_{p_e}^{\infty} U(p(\theta))\, f(p(\theta))dp(\theta) + e^{-r\theta} \int_{-\infty}^{p_e} zI\, f(p(\theta))dp(\theta) \tag{6}$$

where $f(p(\theta))$ is the probability density function of $p(\theta)$.

Using the moment generating function (see Appendix of Bar-Ilan and Strange, 1996), we can simplify Eq. (6) to

$$\begin{aligned} V(p, \theta) = {} & [1 - \Phi(u - \gamma_2\sigma\sqrt{\theta})]Ap^{\gamma_2} \\ & + [1 - \Phi(u - \sigma\sqrt{\theta})]\frac{pe^{-(r-\mu)\theta}}{(r - \mu)} \\ & + \Phi(u)\left(zI + \frac{w}{r}\right)e^{-r\theta} - \frac{w}{r}e^{-r\theta} \end{aligned} \tag{7}$$

where the constant $A$ is given by Eq. (5), $u = \frac{\log(p_e) - \log(p) - (\mu - \sigma^2/2)\theta}{\sigma\sqrt{\theta}}$, and $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Thus, if the investment is made when the output price is $p$, the expected payoff at that time is $V(p, \theta)$ in Eq. (7).

The optimal investment trigger $p^*$ is the solution to the option exercise problem (for the option to invest in the project). The value of this option is given by

$$F(p) = B_1 p^{\gamma_1}$$

where $B_1$ is a constant, and $\gamma_1(>1)$ is given by

$$\gamma_1 = 0.5 - \mu/\sigma^2 + \sqrt{2r/\sigma^2 + \left(0.5 - \mu/\sigma^2\right)^2}. \tag{8}$$

The option will be exercised when the price rises to $p = p^*$, giving two boundary conditions:

$$\text{Value-matching}: F(p^*) = B_1(p^*)^{\gamma_1} = V(p^*, \theta) - I$$

$$\text{Smooth-pasting}: F'(p^*) = B_1\gamma_1(p^*)^{\gamma_1 - 1} = V_p(p^*, \theta)$$

where $V(p, \theta)$ is given by Eq. (7), and the partial derivative $V_p(p, \theta)$ is obtained by directly differentiating Eq. (7) with respect to $p$:

$$\begin{aligned} V_p(p, \theta) = {} & [1 - \Phi(u - \gamma_2\sigma\sqrt{\theta})]A\gamma_2 p^{\gamma_2 - 1} \\ & + \frac{\varphi(u - \gamma_2\sigma\sqrt{\theta})Ap^{\gamma_2}}{p\sigma\sqrt{\theta}} \\ & + [1 - \Phi(u - \sigma\sqrt{\theta})]\frac{e^{-(r-\mu)\theta}}{(r - \mu)} \\ & + \varphi(u - \sigma\sqrt{\theta})\frac{e^{-(r-\mu)\theta}}{(r - \mu)\sigma\sqrt{\theta}} \\ & - \frac{\varphi(u)(zI + w/r)e^{-r\theta}}{p\sigma\sqrt{\theta}} \end{aligned} \tag{9}$$

where $\phi(\cdot)$ represents the pdf of the standard normal distribution. The boundary conditions give the following.

**Proposition 1.** *With an implementation lag of $\theta$, the optimal investment trigger $p^*$ is given by the solution to the equation:*

$$V(p^*, \theta) - I = p^* V_p(p^*, \theta)/\gamma_1 \tag{10}$$

where $V(p, \theta)$ and $V_p(p, \theta)$ are given by Eqs. (7) and (9) respectively.

The existence and uniqueness of the solution in Proposition 1 are discussed in the Appendix.