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An alternative axiomatization of intertemporal utility smoothing

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HIGHLIGHTS

- We axiomatize a model of intertemporal utility smoothing.
- The axiomatization does not require introducing auxiliary consumption risk.
- We derive a particular form of the aggregator function in the recursive utility.

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ABSTRACT

We propose an alternative axiomatization of the model of intertemporal utility smoothing suggested by Wakai (2008) without introducing auxiliary consumption risk.

1. Introduction

Intertemporal choices are often analyzed using the discounted utility model, but experimental studies have generated results that seem to contradict this model. Wakai (2008) proposes the following refinement of the discounted utility model that is consistent with some of those experimental findings. Formally, the utility of an infinite consumption sequence $(c_0, c_1, \ldots)$ is expressed in a recursive form:

$$V(c_0, c_1, \ldots) = \min_{\delta \in [\underline{\delta}, \overline{\delta}]} \{(1-\delta)u(c_0) + \delta V(c_1, c_2, \ldots)\}, \tag{1}$$

where u is the instantaneous utility function and $\underline{\delta}$ and $\overline{\delta}$ are the upper and lower bounds of discount factors satisfying $0 < \underline{\delta} \leq \overline{\delta} < 0$. Representation (1) captures the notion of intertemporal utility smoothing – that is, a desire to lower volatility in a utility sequence – via the following form of recursive gain/loss asymmetry: (i) current utility $u(c_0)$ becomes a reference point to evaluate future utility $V(c_1, c_2, \ldots)$, and (ii) the difference between future utility $V(c_1, c_2, \ldots)$ and current utility $u(c_0)$ defines a gain or a loss, and gains are discounted more than losses. In particular, if $\underline{\delta} = \overline{\delta}$, representation (1) coincides with the discounted utility model.

On theoretical grounds, Koopmans (1960) provides an axiomatization for the discounted utility model on the preference domain that consists of deterministic consumption sequences. In particular, he derives the instantaneous utility function u based on time-separability of preferences. This means that the technique used in Koopmans (1960) cannot be adopted to derive the instantaneous utility function u in representation (1) because representation (1) is non-time-separable. Therefore, Wakai (2008) introduces auxiliary consumption risk because such devices allow us to derive the instantaneous utility function u in a simple axiomatic system.[1] However, the introduction of auxiliary randomization devices comes with a cost: he must impose an assumption that is inconsistent with the way intertemporal consumption risk resolves over time.

Given the above problem, the objective of this paper is to axiomatize representation (1) on the set of deterministic consumption sequences without introducing auxiliary consumption risk.

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[1] To model a dislike of utility variations between adjacent periods, Gilboa (1989) and Shalev (1997) adopted the same domain as Wakai (2008).

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We achieve this goal by deriving a particular form of the aggregator function in the framework of Koopmans (1960) recursive utility. More specifically, we adopt the notion of biseparable preferences as developed by Ghirardato and Marinacci (2001) in the context of choice under uncertainty on a Savage (1954) domain by reinterpreting time periods as possible states of the world. Biseparable preferences describe the behavior of two-dimensional choice problems, which is represented by the function that is a weighted average of cardinal vNM utility indices under a rank-dependent weighting function. In Koopmans (1960) recursive utility, the stationarity axiom reduces the intertemporal choice problem to a two-dimensional choice problem with the dimensions being utility today versus continuation utility from tomorrow onward. Thus, the adoption of biseparable preferences leads to a representation that is a weighted average of today's utility and continuation utility under a rank-dependent weighting function. We then impose an axiom that captures the notion of intertemporal utility smoothing and derive a particular form of weighting function as shown in representation (1). Finally, we solve the recursive equation (1) and derive the explicit functional form of representation V.

The remainder of the paper presents sets of axioms and representations. We also provide proofs that show the equivalence between these axioms and representations.

2. Representation

We consider an infinite-horizon, discrete-time model, where time varies over $\{0, 1, 2, \ldots\} = \mathbb{N}$. The axiomatization exhibited below can be easily adapted to a finite-horizon, discrete-time model with a minor modification. A decision maker (DM) consumes a single perishable good at each period $t \in \mathbb{N}$ from a connected and compact set $X = [\underline{x}, \overline{x}] \subset \mathbb{R}_{++}$, where $\overline{x} > \underline{x}$. We denote a set of deterministic consumption sequences by

$$Y \equiv \{(c_0, c_1, \ldots) \in \mathbb{R}^{\infty} | c_t \in X \text{ for each } t \in \mathbb{N}\},$$

which is endowed with the product topology. Let $\langle c_t \rangle$ be a generic element of Y, where $\langle c_t \rangle = (c_0, c_1, \ldots)$. Let C be the set of all constant deterministic consumption sequences, where a generic element of C is denoted by $\langle c \rangle^* = (c, c, c, \ldots)$. The DM faces the same choice set Y at each time t, and the DM's preference ordering on Y, denoted by $\succeq$, is assumed to be complete, transitive, continuous, nondegenerate, and independent of time and the payoff history.

We first assume the following axioms that characterize the recursive utility.

Axiom 1 (*Atemporal Preference (AP)*). For all $\langle c_t \rangle, \langle c'_t \rangle \in Y$ and $x, x' \in X$, $(x, \langle c_t \rangle) \succeq (x', \langle c_t \rangle)$ if and only if $(x, \langle c'_t \rangle) \succeq (x', \langle c'_t \rangle)$.

Axiom 2 (*Stationarity (ST)*). For all $(x, \langle c_t \rangle)$ and $(x, \langle c'_t \rangle) \in X \times Y$, $(x, \langle c_t \rangle) \succeq (x, \langle c'_t \rangle)$ if and only if $\langle c_t \rangle \succeq \langle c'_t \rangle$.

Koopmans (1960) introduced these axioms, which are also a part of assumptions that characterize the discounted utility model (AP is Postulate (3a) and ST is a combination of Postulate (3b) and Postulate (4)). In particular, AP induces the ordering on X, which is independent of a continuation payoff $\langle c_t \rangle$: for $x, x' \in X$, $x \succeq x'$ if and only if $(x, \langle c_t \rangle) \succeq (x', \langle c_t \rangle)$, where $\langle c_t \rangle$ is any element in Y.[2] Furthermore, ST assumes that the passage of time does not alter the preference ordering, which induces a dynamically consistent decision process.

In terms of the relationship between the ordering on X and the ordering on Y, much of the literature assumes the following form of monotonicity.

Monotonicity. *For any $\langle c_t \rangle, \langle c'_t \rangle \in Y$, if $c_t \succeq c'_t$ for all $t \in \mathbb{N}$, then $\langle c_t \rangle \succeq \langle c'_t \rangle$. The latter ranking is strict if the former ranking is strict for some $t \in \mathbb{N}$.*

The following result can be easily derived so we state it without a proof.

Lemma 1. *Given continuity, Axioms 1 and 2 imply monotonicity.*

Because X is connected and compact, it follows from Lemma 1 and continuity that, for each $\langle c_t \rangle \in Y$, there exists $\langle x \rangle^* \in C$ such that $\langle x \rangle^* \simeq \langle c_t \rangle$. We call this x a *constant equivalent of* $\langle c_t \rangle$ and refer to it as $ce(\langle c_t \rangle)$.

To model the recursive gain/loss asymmetry, we must first derive asymmetric weights for gains versus losses as well as the instantaneous utility function u. Thus, we consider a *binary sequence* $\langle x : y \rangle$, which is a consumption sequence $\langle c_t \rangle \in Y$ such that $c_t = x \in X$ for $t = 0$ and $c_t = y \in X$ for $t \geq 1$. Let Y_b be the collection of all binary sequences, each element of which, as shown above, is either increasing or decreasing. Furthermore, for $\langle c_t \rangle, \langle c'_t \rangle \in Y_b$, the *mixture of* $\langle c_t \rangle$ *and* $\langle c'_t \rangle$ is the binary sequence in Y_b, denoted by $\langle \langle c_t \rangle : \langle c'_t \rangle \rangle$, such that, for each $\tau \in \mathbb{N}$, $\langle \langle c_t \rangle : \langle c'_t \rangle \rangle_\tau = ce(\langle c_\tau : c'_\tau \rangle)$. Thus, by monotonicity, for each $\tau \in \mathbb{N}$, $c_\tau \succeq \langle \langle c_t \rangle : \langle c'_t \rangle \rangle_\tau \succeq c'_\tau$ if $c_\tau \succeq c'_\tau$, and $c'_\tau \succeq \langle \langle c_t \rangle : \langle c'_t \rangle \rangle_\tau \succeq c_\tau$ if $c'_\tau \succeq c_\tau$. Moreover, we also state that $\langle c_t \rangle$ and $\langle c'_t \rangle$ are comonotonic if there are no $\tau, \tau' \in \mathbb{N}$ such that $c_\tau \succ c_{\tau'}$ and $c'_{\tau'} \succ c'_\tau$.

Recursive gain/loss asymmetry implies that comonotonic consumption sequences in Y_b are evaluated under the same decision weight. To capture this idea, we adopt the following version of the independence axiom on Y_b from Ghirardato and Marinacci (2001), which is suitably modified to fit our framework, where $\{x, y\} \succeq z$ stands for $x \succeq z$ and $y \succeq z$.

Axiom 3 (*Comonotonic Independence for Binary Consumption Sequences (CI)*). For all $\langle x : y \rangle, \langle x' : y' \rangle, \langle x'' : y'' \rangle \in Y_b$ that are pairwise comonotonic, if $\{x, x'\} \succeq x''$ and $\{y, y'\} \succeq y''$ (or $x'' \succeq \{x, x'\}$ and $y'' \succeq \{y, y'\}$), then $\langle x : y \rangle \succeq \langle x' : y' \rangle$ implies $\langle \langle x : y \rangle : \langle x'' : y'' \rangle \rangle \succeq \langle \langle x' : y' \rangle : \langle x'' : y'' \rangle \rangle$ and $\langle \langle x'' : y'' \rangle : \langle x : y \rangle \rangle \succeq \langle \langle x'' : y'' \rangle : \langle x' : y' \rangle \rangle$.[3]

CI states that among the comonotonic consumption sequences in Y_b satisfying the stated condition, the mixture operation does not alter the preference ordering. The required condition is that the mixture must be taken with a dominated (or dominating) consumption sequence because such an operation guarantees that $\langle x : y \rangle$, $\langle x' : y' \rangle$, and a mixture of $\langle x : y \rangle$ or $\langle x' : y' \rangle$ with $\langle x'' : y'' \rangle$ are all pairwise comonotonic.

The above axiom leads to the following lemma.

Lemma 2. *Assume that $\succeq$ satisfies Axioms 1 and 2. Then the following statements are equivalent.*

(i) *$\succeq$ satisfies Axiom 3.*

(ii) *There exists a continuous and nontrivial function $u : X \to \mathbb{R}$, real numbers $\underline{\delta}$ and $\overline{\delta}$ satisfying $0 < \underline{\delta}, \overline{\delta} < 1$ such that $\succeq$ on Y_b is represented by $F : Y_b \to \mathbb{R}$, where*

$$F(\langle x : y \rangle) \equiv \begin{cases} (1 - \overline{\delta})u(x) + \overline{\delta}u(y) & \text{if } x \succeq y \\ (1 - \underline{\delta})u(x) + \underline{\delta}u(y) & \text{if } x \preceq y. \end{cases} \tag{2}$$

Moreover, $\underline{\delta}$ and $\overline{\delta}$ are unique, and u is unique up to a positive affine transformation.

[2] The literature regarding risk and uncertainty often defines the induced ordering on the consumption set X as follows: for $x, y \in X$, $x \succeq y$ if and only if $\langle x \rangle^* \succeq \langle y \rangle^*$, where $\langle x \rangle^*$ and $\langle y \rangle^*$ are acts that pay x and y at every state, respectively. This definition lacks a behavioral foundation in an intertemporal setting because consuming x at each period is not identical to consuming x in a single period.

[3] This is a simplified version of the Binary Comonotonic Act Independence axiom used in Ghirardato and Marinacci (2001).