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Asymptotic and bootstrap inference for top income shares

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HIGHLIGHTS

- We use Monte Carlo methods to study statistical inference for top income shares.
- Asymptotic inference performs poorly even in large samples.
- Standard bootstrap methods perform better but can be unreliable.
- Semi-parametric bootstrap gives accurate inference in moderate samples.
- Asymptotic inference should be avoided in practice.

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ABSTRACT

We analyse statistical inference for top income shares in finite samples. The asymptotic inference performs poorly even in large samples. The standard bootstrap tests give some improvement, but can be unreliable. The semi-parametric bootstrap approach is accurate in moderate and larger samples.

1. Introduction

In recent years, there has been growing interest in economic analysis of top income shares, that is, the shares of total income held by the richest x% (e.g., 10%, 5% or 1%) of the population. Long-run time series of top income shares for more than twenty countries have been constructed (Atkinson and Piketty, 2007, 2010; Atkinson et al., 2011), which facilitates the study of trends in income distributions that spans decades and, in some cases, the entire 20th century. Although much of this literature uses income tax data, several contributions are also based on household survey data. The latter are used to estimate top income shares when tax return data are not available or only available for a limited time span (see, e.g. Leigh and Van der Eng, 2009; Piketty and Qian, 2009) or when one wants to compare results from tax returns and survey data (Burkhauser et al., 2012).

Income tax data used in the recent literature on top income shares are usually based on very large samples, sometimes covering the whole population of taxpayers. On the other hand, household survey data often come from samples of moderate or rather small size. In this context, the question of sampling variability of top income shares calculated using survey data arises naturally. Although asymptotic inference methods for income shares exist since 1980s (Beach and Davidson, 1983), their performance in finite samples has not been studied. Moreover, as shown recently by Davidson and Flachaire (2007) asymptotic and standard bootstrap inferences for some popular inequality measures such as the Theil index are unreliable even in very large samples.[1] They argued that the reason for the poor performance

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[1] See also Cowell and Flachaire (2007), Davidson (2012) and Schluter (2012) for other contributions to this literature.

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of asymptotic and standard bootstrap methods is the sensitivity of some inequality indices to the exact nature of the upper tail of the income distribution. Since top income shares are constructed as ratios of (portions of) upper tails to total income of a population, it may be expected that the problems observed by Davidson and Flachaire (2007) are equally or even more relevant for top income shares.

This paper uses Monte Carlo simulations to study the finite-sample performance of asymptotic inference for top income shares. We also compare the asymptotic inference with the standard bootstrap and a non-standard semi-parametric bootstrap method proposed by Davidson and Flachaire (2007). We find that the asymptotic approach fails in providing accurate inference, while the performance of standard bootstrap is better but not satisfactory. Monte Carlo results suggest that the semi-parametric bootstrap performs well in moderate samples.

2. Asymptotic variance estimation for top income shares

The asymptotic approach to variance estimation for quantile shares was introduced by Beach and Davidson (1983), who derived the distribution-free asymptotic joint variance–covariance structure for income quantile shares and Lorenz curve ordinates (cumulative income shares).[2] Let $X = (x_1, \ldots, x_N)$ be an income distribution with the cumulative distribution function (CDF) F, mean income μ and variance λ^2. An income quantile of order p, ξ_p, with $0 < p < 1$, is defined implicitly by $F(\xi_p) = p$. Quantiles divide the sample into $n = 1/p$ quantile groups; $p_i = i/n$, $i = 1, \ldots, n-1$, denote the quantile proportions. In the case of deciles, there are $K = 9$ quantiles with quantile proportions $p_1 = 0.1, \ldots, p_9 = 0.9$. The top quantile share is defined as

$$\psi_T = 1 - \Phi(\xi_K), \tag{1}$$

where $\Phi(\xi_K)$ is a population Lorenz curve ordinate for income quantile K, defined as

$$\Phi(\xi_K) = \frac{1}{\mu}\int_0^{\xi_K} x dF(x) = p_K \frac{\gamma_K}{\mu} \tag{2}$$

where γ_K is the conditional mean of incomes less than or equal to ξ_K. The top income share (1) can be estimated using a plug-in estimator as

$$\hat{\psi}_T = 1 - p_K \frac{\hat{\gamma}_K}{\hat{\mu}}. \tag{3}$$

Distribution-free asymptotic variance of top quantile shares is derived by Beach and Davidson (1983) as

$$Var(\psi_T) = \frac{1}{N}\frac{p_K}{\mu^2}\left[\lambda^2 \frac{p_K \gamma_K^2}{\mu^2} + \lambda_K^2\left(1 - \frac{p_K \gamma_K}{\mu}\right) + (1 - p_K) \right.$$
$$\left. \times (\xi_K - \gamma_K)^2 - 2\frac{p_K \gamma_K}{\mu}(\xi_K - \gamma_K)(\mu - \gamma_K)\right], \tag{4}$$

where λ_K^2 is the conditional variance of incomes less than or equal to ξ_K. It can be estimated by replacing $\mu, \gamma_K, \lambda^2, \lambda_K^2$, and ξ_K with their sample estimates.

3. Asymptotic and bootstrap tests

In order to test a hypothesis that $\psi_T = \psi_T^0$, for some value of ψ_T^0, we can use estimate of the top income share, $\hat{\psi}_T$, given by (3) and estimate of its variance, $\hat{V}ar(\hat{\psi}_T)$ from (4) and construct the following asymptotic t-type statistic

$$W = \frac{\hat{\psi}_T - \psi_T^0}{[\hat{V}ar(\hat{\psi}_T)]^{0.5}}. \tag{5}$$

In our simulations, we use data drawn from the 4-parameter Generalized Beta of the Second Kind (GB2) distribution, which is widely used in modelling earnings and income distributions and known to fit these distributions very well (McDonald, 1984; Kleiber and Kotz, 2003; Dastrup et al., 2007). The GB2 distribution is defined by its CDF as

$$F(x) = I\left(p, q, \frac{(x/b)^a}{1 + (x/b)^a}\right), \quad x > 0 \tag{6}$$

where parameters a, b, p, q are each positive and $I(p, q, x)$ is the regularized incomplete beta function. See Kleiber and Kotz (2003) for a detailed review of the GB2 properties. The sample sizes in our simulations are $n = 500, 1000, 2000, 3000, 4000, 5000$, and $10\,000$.[3] We use the GB2 parameter values $a = 2.2474$, $b = 58\,441.5$, $p = 0.6186$ and $q = 1.118$, taken from McDonald and Ransom (2008), who have shown that the GB2 distribution is the best choice for fitting 2000 family income data for the USA. Butler and McDonald (1989) introduce "normalized incomplete moments" for the GB2 distribution, which can be used to calculate income shares received by quantiles. For our choice of parameters, the "true" values of the top 10%, 5% and 1% income shares that we analyse in our simulations are 0.345895, 0.231211 and 0.088844, respectively. We calculate an asymptotic P value from (5) replacing ψ_T^0 by an appropriate "true" value and using the Student distribution with n degrees of freedom.

We compare the performance of asymptotic inference with standard bootstrap method known as the percentile-t or bootstrap-t method. The test is constructed as follows. First, we compute W statistics as given by (5) from the original sample and then we draw B bootstrap samples of the same size as the original sample. We set B to 199 in our simulation. For each bootstrap sample $j, j = 1, \ldots, 199$, we compute W_j^* statistic in the same way as W was computed from the original sample, but with ψ_T^0 replaced by the index $\hat{\psi}_T$ estimated from the original sample. The bootstrap P value is the proportion of the bootstrap samples for which the bootstrap statistic W_j^* is more extreme than W.

We complement the asymptotic and standard bootstrap tests with a non-standard bootstrap test proposed by Davidson and Flachaire (2007) in the context of improving inference for the Theil index of inequality. Their semi-parametric bootstrap test combines parametric modelling of the upper tail with the standard non-parametric bootstrap for the rest of the distribution. The test is implemented as follows. First, the W statistic as defined in (5) is computed for the original sample of size n. Second, a Pareto distribution is fitted to the c largest incomes with c chosen using a procedure introduced by Dupuis and Victoria-Feser (2006). Third, the "true" value of a top quantile share, ψ_T^*, is computed for a semi-parametric distribution consisting of the estimated Pareto tail and the sample values for the rest of the distribution. Fourth, a bootstrap sample is drawn with probability $p_t = c/n$ from the Pareto distribution and with probability $1 - p_t$ from the $n(1 - p_t)$ smallest observations from the original sample. Fifth, for the bootstrap sample the bootstrap statistic W^* is computed using (5) with ψ_T^0 replaced by ψ_T^*. The last two steps of this procedure are repeated 199 times. The semi-parametric bootstrap Pvalue is the proportion of W_j^*, $j = 1, \ldots, 199$, that are more extreme than W.

[2] See also Beach and Kaliski (1986) for inference for income shares with sample weights. Binder and Kovacevic (1995) and Verma and Betti (2011) use other approaches to derive asymptotic inference for income shares.

[3] We do not provide results for smaller sample sizes because when $n < 500$ the number of observations needed to calculate the top incomes, especially belonging to the top 1%, is too small to allow for a meaningful variance estimation of ψ_T.