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Matching with quorums[☆]

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HIGHLIGHTS

- We consider the problem of allocating agents to projects that have a minimum quorum.
- The serial dictatorship mechanism is not efficient or strategy proof.
- We propose a mechanism: serial dictatorship with project closures.
- The set of available projects evolves so that already-chosen projects are not closed.
- Our mechanism is strategy proof and efficient.

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ABSTRACT

In the problem of allocating workers to different projects, where each project needs a minimum number of workers assigned to it, the serial dictatorship mechanism is neither strategy proof nor Pareto efficient. We therefore propose a strategy-proof and Pareto-efficient mechanism.

1. Introduction

We consider the problem of assigning agents to different projects that have a minimum quorum and a maximum capacity. Firms with multiple projects routinely face this problem: they must decide how to better allocate the workforce among different projects, and each worker must be allocated to at most one project. In addition, projects typically require a minimum number of workers in order to be completed successfully; hence, firms do not initiate a given project if the minimum quorum is not satisfied. This could be the case, for example, for projects that have a large fixed cost or that present economies of scale. At the same time, since allocating too many workers to a project is inefficient, the firms may require a maximal capacity for each project. Some educational institutions face a similar problem when assigning students to classes. Students must choose which classes to take in a given semester, during which there are many potential course offerings. Once all students registered for their classes – some of which are not mandatory – the courses will be offered only if a minimum quorum is satisfied.[1] At the same time, there is a limit on the class size due to physical space restrictions.

First we show that, in our setting, the well-known serial dictatorship mechanism is neither Pareto efficient nor strategy proof, regardless of how the agents are ordered. This is because agents who make the initial selections must consider the possibility that

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[1] A director of graduate studies might find this problem to be a familiar one. In fact, as anecdotal evidence, some Ph.D. programs in the US regularly face this situation, in which a course will not be offered unless a predetermined minimal enrollment is met.

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the projects they select may never get opened due to insufficient enrollment. Consequently, agents might want to choose a less preferred project with a lower quorum. Motivated by this problem, we propose a strategy-proof and Pareto-efficient mechanism which we call *serial dictatorship with project closures*. Our mechanism is a stronger form of serial dictatorship in that the set of projects from which an agent can choose evolves so that already-chosen projects are opened.

The serial dictatorship mechanism in problems without any minimum quorum restriction satisfies many positive properties. In the house allocation setting, Svensson (1999) shows that it is the only deterministic mechanism that is strategy proof, nonbossy and neutral.[2] Abdulkadiroglu and Sönmez (1998) show that, for each Pareto-efficient allocation, there exists an ordering of the agents such that the serial dictatorship mechanism delivers the allocation. Furthermore, the core from random endowments is equivalent to the random serial dictatorship,[3] which provides an additional justification for the use of the random serial dictatorship in practice.

The study of matching problems with a minimum quorum is recent: Hamada et al. (2008) and Biró et al. (2010) study two-sided matching problems with a quorum in which both sides have well-defined preferences. They concentrate on stability, and show that stable matchings do not necessarily exist. The question of how to find a stable matching, if indeed it exists, is still under study. Meanwhile, the current paper studies efficiency and strategy-proofness.

Manea (2007) considers environments in which agents want to consume more than one object, and he studies a weak form of serial dictatorship: agents choose one object at a time according to an ordering in which any given agent could appear more than once. He shows that, in such environments, this weak version of the serial dictatorship mechanism fails both strategy-proofness and efficiency. In fact, Pápai (2001), Ehlers and Klaus (2003), and Hatfield (2009) establish a general result in such environments. The only strategy-proof Pareto-optimal and nonbossy mechanisms are the strong form of the sequential dictatorship: each agent chooses his/her favorite set of available objects according to a predefined ordering. In contrast, in our setting each agent is entitled to only one object, yet the serial dictatorship mechanism still fails in terms of efficiency and strategy-proofness.

2. Model

Finite set $I = \{1, \ldots, n\}$ is the set of agents/workers, and finite set $P = \{p_1, \ldots, p_m\}$ is the set of projects. Each project $p \in P$ has a maximum capacity k_p, with $1 \leq k_p \leq \infty$, and a minimum quorum $q_p \geq 1$. Both k_p and q_p are integer numbers, and throughout the paper we assume that $q_p \leq k_p$. This means that each project p cannot have more than k_p agents assigned to it. In addition, any project p with fewer than q_p agents assigned to it does not open. For convenience, we assume that $q_p \leq n$ for all $p \in P$; otherwise, project p would never open. Let $k = (k_p)_{p\in P}$ and $q = (q_p)_{p\in P}$.

Each agent $i \in I$ has a preference ordering $\succeq_i$ over the projects and the empty set $\emptyset$. The preference profile $(\succeq_i)_{i\in I}$ will be denoted by $\succeq$. Let $\succ_i$ and $\sim_i$ be the respective strict and indifference relations associated with $\succeq_i$. Throughout the paper, we maintain two assumptions about preferences: (1) each player's preference ordering $\succeq_i$ is strict, i.e., $p \succeq_i p'$ if either $p \succ_i p'$ or $p = p'$, and (2) $p \succ_i \emptyset$ for all $i \in I$ and $p \in P$; that is, every project is acceptable to any agent.

[2] The house allocation problem was first studied by Hylland and Zeckhauser (1979).

[3] This is a serial dictatorship in which the ordering is the outcome of a lottery.

A matching μ is a correspondence $\mu : I \cup P \to I \cup P$ such that (i) $\mu(i) \subseteq P$, for all $i \in I$; (ii) $\mu(p) \subseteq I$, for all $p \in P$; and (iii) $p \in \mu(i)$ if and only if $i \in \mu(p)$. If $\mu(i) = \emptyset$, we say that i is unmatched or unassigned at μ or that i is not assigned to any project at μ. Similarly, if $\mu(p) = \emptyset$, we say that p is unmatched at μ. For simplicity, we will write $\mu(i) = p$ instead of $\mu(i) = \{p\}$.

We will concentrate on matchings that assign each agent i to at most one project and each project p to at least q_p and at most k_p agents.

Definition 1 (*Feasible Matching*). A matching μ is feasible if the following two conditions are satisfied:

(i) for all $p \in P$, either $q_p \leq |\mu(p)| \leq k_p$ or $|\mu(p)| = 0$; and
(ii) $|\mu(i)| \in \{0, 1\}$ for all $i \in I$.

According to our assumption that every project is acceptable to any agent, each feasible matching μ is individually rational; i.e., if $\mu(i) = p$, then $p \succ_i \emptyset$, for all $i \in I$. The definition of Pareto efficiency in our setting coincides with the standard one.

Definition 2 (*Pareto Efficiency*). A *feasible* matching $\bar{\mu}$ Pareto dominates a feasible matching μ if

$$\bar{\mu}(i) \succ_i \mu(i) \quad \text{for at least one } i \in I \quad \text{and}$$
$$\bar{\mu}(j) \succeq_j \mu(j), \quad \text{for } \forall j \in I.$$

A matching μ is Pareto efficient if it is feasible and, in addition, there does not exist any feasible matching $\bar{\mu}$ that Pareto dominates μ.

A matching market M with quorums is given by the set of agents, the set of projects, the quorums and capacities of the projects, and the agents' preference profiles, i.e., $M = (I, P, q, k, \succeq)$. A mechanism φ for such markets is a mapping that assigns a feasible matching for each market.

A mechanism is Pareto efficient if it results in a Pareto- efficient matching for each market. Below, we define strategy-proofness.

Definition 3 (*Strategy-Proofness*). Let φ be a mechanism for the set of matching markets with quorums. We say that φ is *manipulable* (individually) if there exist two markets $M = (I, P, k, q, \succeq)$ and $M' = (I, P, k, q, \succeq')$ and $i \in I$ such that (i) M' differs from M only in agent i's preference ordering (i.e., $\succeq_i \neq \succeq'_i$ and $\succeq_j = \succeq'_j$, for all $j \neq i$), and (ii) $\mu'(i) \succ_i \mu(i)$, where $\mu \equiv \varphi(M)$ and $\mu' \equiv \varphi(M')$. A mechanism φ is strategy proof if it is not manipulable.

3. Serial dictatorship

The algorithm known as serial dictatorship (SD) has been widely used in matching problems, both in theory and in practice. In environments without the minimum quorum restriction, the SD algorithm with respect to a given ordering of the agents in I is applied to market M as follows. Following the ordering of the agents, each agent is sequentially assigned to his/her most preferred project (with respect to his/her preferences in market M) among those projects that have not yet reached their maximum capacities. The algorithm terminates once the last agent in the ordering is assigned, or when all projects have reached their maximum capacities. The SD mechanism is the revelation mechanism which maps each market M to the matching produced by the SD algorithm for market M. This mechanism has been shown to be Pareto efficient and strategy proof (see, for example, Svensson, 1999).

In contrast, if we apply the SD algorithm to a market which has a minimum quorum restriction, the resulting matching may not be feasible. Specifically, there may be projects for which the number of agents assigned is lower than their minimum quorum. Therefore, for simplicity, we assume that, in the last step of the SD