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On the estimation and inference in factor-augmented panel regressions with correlated loadings

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HIGHLIGHTS

- The paper considers the very popular problem of estimating factor-augmented regressions.
- The Pesaran (2006) approach considered has attracted much attention in the literature.
- We focus on the uncorrelated loadings assumption that is not made explicit in Pesaran (2006).
- We demonstrate how correlated loadings can render the approach inconsistent.

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ABSTRACT

In an influential paper, Pesaran [Pesaran, M.H. (2006). Estimation and inference in large heterogeneous panels with a multifactor error structure. Econometrica 74, 967–1012] proposes a very simple estimator of factor-augmented regressions that has since then become very popular. In this note we demonstrate how the presence of correlated factor loadings can render this estimator inconsistent.

1. Introduction

One of the first approaches to accommodate error cross-section dependence in the form of common factors when estimating static panel data regressions is that of Coakley et al. (2002), which augments the original regression with estimated principal component (PC) factors obtained from the residuals of a preliminary first-step OLS fit.[1] However, as Pesaran (2006) shows, this estimator will not be consistent if the factors and the included regressors are correlated, as this will render the resulting first-step OLS estimator inconsistent.

As a possible solution to the above problem Pesaran (2006) proposes augmenting the regression not with estimated PC factors, but rather with the cross-section averages of the observables. The resulting common correlated effects (CCE) approach removes the need for an initial consistent estimate, and is therefore consistent even if the factors are correlated with the regressors. Despite its simplicity, the CCE approach has shown to be very general when it comes to the types of cross-section dependence that can be accommodated (see Chudik et al., 2011), and does not even require the number of factors to be smaller than the number of cross-section averages that are being used in their stead. The approach has also been shown to perform relatively well in small samples (see Kapetanios and Pesaran, 2005; Chudik et al., 2011), and has been generalized in a number of directions (see, for example Kapetanios et al., 2011).

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[1] See Baltagi (2008) for a textbook exposure of panel data estimation and inference.

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In the present note we demonstrate how CCE relies on the factor loadings of the regression error and regressors being uncorrelated, an assumption that is discussed at some length in Bai (2009) but that is not made explicit in Pesaran (2006), and that a violation will render the approach inconsistent. We also show how the problem with correlated loadings is not easily circumvented, at least not without making other "compensating" assumptions. One possibility in this regard is to assume that the number of cross-section averages are at least as large as the number of factors.

2. Model

The data generating process (DGP) that we consider is the same as in Pesaran (2006) and is given by

$$\mathbf{y}_i = \mathbf{D}\boldsymbol{\alpha}_i + \mathbf{X}_i\boldsymbol{\beta}_i + \mathbf{F}\boldsymbol{\gamma}_i + \boldsymbol{\varepsilon}_i, \tag{1}$$

$$\mathbf{X}_i = \mathbf{D}\mathbf{A}_i + \mathbf{F}\boldsymbol{\Gamma}_i + \mathbf{V}_i, \tag{2}$$

or, equivalently,

$$\mathbf{Z}_i = \mathbf{D}\mathbf{B}_i + \mathbf{F}\mathbf{C}_i + \mathbf{u}_i, \tag{3}$$

where $i = 1, \ldots, N$, $\mathbf{y}_i = (y_{i1}, \ldots, y_{iT})'$ and $\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})'$ are $T \times 1$, $\mathbf{D} = (\mathbf{d}_1, \ldots, \mathbf{d}_T)'$ is $T \times n$, $\mathbf{X}_i = (\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT})'$ and $\mathbf{V}_i = (\mathbf{v}_{i1}, \ldots, \mathbf{v}_{iT})'$ are $T \times k$, $\mathbf{F} = (\mathbf{f}_1, \ldots, \mathbf{f}_T)'$ is $T \times m$, $\mathbf{Z}_i = (\mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT})'$ is $T \times (m+1)$, $\mathbf{z}_{it} = (y_{it}, \mathbf{x}_{it}')'$, $\mathbf{B}_i = (\boldsymbol{\alpha}_i + \mathbf{A}_i\boldsymbol{\beta}_i, \mathbf{A}_i)$, $\mathbf{C}_i = (\boldsymbol{\gamma}_i + \boldsymbol{\Gamma}_i\boldsymbol{\beta}_i, \boldsymbol{\Gamma}_i)$, $\mathbf{u}_i = (\mathbf{u}_{i1}, \ldots, \mathbf{u}_{iT})'$ and $\mathbf{u}_{it} = (\varepsilon_{it} + \mathbf{v}_{it}'\boldsymbol{\beta}_i, \mathbf{v}_{it}')'$. $\mathbf{D}$ and $\mathbf{F}$ are observed and latent common effects, respectively, that are assumed to be covariance stationary (see Pesaran, 2006, Assumption 1). The idiosyncratic errors $\boldsymbol{\varepsilon}_i$ and $\mathbf{V}_i$ are mean zero, and independent of each other and also across i, but where the individual elements, $\boldsymbol{\varepsilon}_{it}$ and $\mathbf{v}_{it}$, may be correlated across t (see Pesaran, 2006, Assumption 2). $\boldsymbol{\beta}_i$, $\boldsymbol{\gamma}_i$ and $\boldsymbol{\Gamma}_i$ are assumed to be random coefficients such that $\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i$, $\boldsymbol{\gamma}_i = \boldsymbol{\gamma} + \boldsymbol{\eta}_i$ and $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma} + \boldsymbol{\epsilon}_i$, where $\boldsymbol{\nu}_i$ and $(\boldsymbol{\eta}_i, \boldsymbol{\epsilon}_i)$ are i.i.d. across i and also independent of all the other random elements of the model (see Pesaran, 2006, Assumptions 3 and 4). All necessary moments are assumed to exist. In particular, the covariance matrices $\boldsymbol{\Omega}_\nu$, $\boldsymbol{\Omega}_\eta$ and $\boldsymbol{\Omega}_\epsilon$ of $\boldsymbol{\nu}_i$, $\boldsymbol{\eta}_i$ and $\boldsymbol{\epsilon}_i$, respectively, are all positive definite.

Define $\bar{\mathbf{Y}}_w = \sum_{i=1}^N w_i\mathbf{Y}_i$ for any variable $\mathbf{Y}_i$, and weights $w_1, \ldots, w_N$ such that $w_i = O(1/N)$ and $\sum_{i=1}^N w_i = 1$ (see Pesaran, 2006, Assumption 5). The CCE approach is based on using $\bar{\mathbf{Z}}_w$ to approximate $\mathbf{F}$. Assume for simplicity that $\mathbf{B}_i = \mathbf{0}$. Since $\mathbf{u}_i$ is independent across i, we have that $\bar{\mathbf{Z}}_w = \mathbf{F}\bar{\mathbf{C}}_w + \bar{\mathbf{u}}_w = \mathbf{F}\bar{\mathbf{C}}_w + O_p(1/\sqrt{N})$, suggesting that as long as

$$rk(\bar{\mathbf{C}}_w) = m \le k+1, \tag{4}$$

$\bar{\mathbf{Z}}_w$ is consistent for the space spanned by $\mathbf{F}$. In this note we follow Pesaran (2006) and focus on the case where the rank condition in (4) is not met, and therefore $\bar{\mathbf{Z}}_w$ need not be consistent. However, as we show in the next section, this creates a dependence on the correlation between the loadings $\boldsymbol{\gamma}_i$ and $\boldsymbol{\Gamma}_i$ measuring the heterogeneous effect of the common shocks on $\mathbf{y}_i$ and $\mathbf{X}_i$, respectively.

3. Theoretical results

While Pesaran (2006) considers both mean group and pooled estimators, here we only consider the latter estimator, defined as

$$\hat{\mathbf{b}}_P = \left(\sum_{i=1}^N \theta_i \mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{X}_i\right)^{-1} \sum_{i=1}^N \theta_i \mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{y}_i,$$

where $\bar{\mathbf{M}}_w = \mathbf{I}_T - \bar{\mathbf{H}}_w(\bar{\mathbf{H}}_w'\bar{\mathbf{H}}_w)^{-1}\bar{\mathbf{H}}_w'$, $\bar{\mathbf{H}}_w = (\mathbf{D}, \bar{\mathbf{Z}}_w)$, and $\theta_1, \ldots, \theta_N$ are weights that satisfy the same conditions as $w_1, \ldots, w_N$ (see Section 2).

We begin by deriving the asymptotic distribution of $\hat{\mathbf{b}}_P$. We then show how this result is affected by the presence of correlation between $\boldsymbol{\gamma}_i$ and $\boldsymbol{\Gamma}_i$. From (B.1) in Pesaran (2006),

$$\begin{aligned}\left(\sum_{i=1}^N \theta_i^2\right)^{-1/2} &(\hat{\mathbf{b}}_P - \boldsymbol{\beta}) \\ &= \left(\sum_{i=1}^N \theta_i \frac{\mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{X}_i}{T}\right)^{-1} \\ &\quad \times \left(\frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{\theta}_i \frac{\mathbf{X}_i'\bar{\mathbf{M}}_w(\mathbf{X}_i\boldsymbol{\nu}_i + \mathbf{F}\boldsymbol{\gamma}_i + \boldsymbol{\varepsilon}_i)}{T} + \mathbf{q}_{NT}\right)\end{aligned}$$

where $\tilde{\theta}_i = \theta_i/\sqrt{\sum_{i=1}^N \theta_i^2/N}$ and

$$\mathbf{q}_{NT} = \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{\theta}_i \frac{(\mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{F})\boldsymbol{\gamma}_i}{T}.$$

Given $\boldsymbol{\gamma}_i = \boldsymbol{\gamma} + \boldsymbol{\eta}_i$, we have $\bar{\boldsymbol{\gamma}}_w$, $\boldsymbol{\gamma}_i = \bar{\boldsymbol{\gamma}}_w + \boldsymbol{\eta}_i - \bar{\boldsymbol{\eta}}_w$. Hence,

$$\begin{aligned}\mathbf{q}_{NT} &= \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{\theta}_i \frac{(\mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{F})}{T} \\ &\quad \times (\bar{\boldsymbol{\gamma}}_w - \bar{\boldsymbol{\eta}}_w) + \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{\theta}_i \frac{(\mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{F})}{T}\boldsymbol{\eta}_i.\end{aligned}$$

If $\theta_i = w_i$, then $\sum_{i=1}^N \tilde{\theta}_i\mathbf{X}_i'\bar{\mathbf{M}}_w = \sum_{i=1}^N \tilde{w}_i\mathbf{X}_i'\bar{\mathbf{M}}_w = \mathbf{0}$, which in turn implies

$$\begin{aligned}\mathbf{q}_{NT} &= \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{w}_i \frac{(\mathbf{X}_i'\bar{\mathbf{M}}_w\mathbf{F})}{T}\boldsymbol{\eta}_i \\ &= \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{w}_i \frac{(\mathbf{X}_i'\bar{\mathbf{M}}_q\mathbf{F})}{T}\boldsymbol{\eta}_i + O_p\left(\frac{1}{\sqrt{N}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right),\end{aligned}$$

where the second equality follows from (38) in Pesaran (2006), $\bar{\mathbf{M}}_q = \mathbf{I}_T - \bar{\mathbf{Q}}_w(\bar{\mathbf{Q}}_w'\bar{\mathbf{Q}}_w)^{+}\bar{\mathbf{Q}}_w'$, $\mathbf{A}^{+}$ is the Moore–Penrose inverse of the matrix $\mathbf{A}$, $\bar{\mathbf{Q}}_w = \mathbf{G}\bar{\mathbf{P}}_w$, $\mathbf{G} = (\mathbf{D}, \mathbf{F})$ and

$$\bar{\mathbf{P}}_w = \begin{pmatrix}\mathbf{I}_n & \bar{\mathbf{B}}_w \\ \mathbf{0} & \bar{\mathbf{C}}_w\end{pmatrix}.$$

Hence, since $\sum_{i=1}^N \tilde{w}_i\mathbf{X}_i'\bar{\mathbf{M}}_w = \mathbf{0}$ and $\sum_{i=1}^N \tilde{w}_i\mathbf{X}_i'\bar{\mathbf{M}}_q\boldsymbol{\varepsilon}_i/\sqrt{NT} = O_p(1)$, and assuming that $\text{plim}_{T\to\infty}\mathbf{X}_i'\bar{\mathbf{M}}_q\mathbf{X}_i/T$ exists and is nonsingular (see Pesaran, 2006, Assumption 5(a)),

$$\begin{aligned}\left(\sum_{i=1}^N w_i^2\right)^{-1/2}(\hat{\mathbf{b}}_P - \boldsymbol{\beta}) &= \left(\sum_{i=1}^N w_i \frac{\mathbf{X}_i'\bar{\mathbf{M}}_q\mathbf{X}_i}{T}\right)^{-1} \\ &\quad \times \frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{w}_i \frac{\mathbf{X}_i'\bar{\mathbf{M}}_q(\mathbf{X}_i\boldsymbol{\nu}_i + \mathbf{F}\boldsymbol{\eta}_i)}{T} \\ &\quad + O_p\left(\frac{1}{\sqrt{N}}\right) + O_p\left(\frac{1}{\sqrt{T}}\right).\end{aligned}$$

If $\boldsymbol{\nu}_i$, $\mathbf{V}_i$, $\boldsymbol{\eta}_i$ and $\boldsymbol{\epsilon}_i$ are mean zero, i.i.d. across i, independent of each other and of $\mathbf{F}$, then $E(\mathbf{X}_i'\bar{\mathbf{M}}_q(\mathbf{X}_i\boldsymbol{\nu}_i + \mathbf{F}\boldsymbol{\eta}_i)) = \mathbf{0}$. For the variance,

$$\begin{aligned}E&\left[\left(\frac{1}{\sqrt{N}}\sum_{i=1}^N \tilde{w}_i \frac{\mathbf{X}_i'\bar{\mathbf{M}}_q(\mathbf{X}_i\boldsymbol{\nu}_i + \mathbf{F}\boldsymbol{\eta}_i)}{T}\right)^2\right] \\ &= E\left(\frac{1}{N}\sum_{i=1}^N\sum_{j=1}^N \tilde{w}_i\tilde{w}_j \frac{\mathbf{X}_i'\bar{\mathbf{M}}_q(\mathbf{X}_j\boldsymbol{\nu}_j + \mathbf{F}\boldsymbol{\eta}_j)}{T}\right. \\ &\quad \left.\times \frac{(\boldsymbol{\nu}_i'\mathbf{X}_i' + \boldsymbol{\eta}_i'\mathbf{F}')\bar{\mathbf{M}}_q\mathbf{X}_j}{T}\right)\end{aligned}$$