Economics Letters 119 (2013) 291–295

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Productivity growth decomposition using a spatial autoregressive frontier model

Anthony Glass *, Karligash Kenjegalieva, Juan Paez-Farrell

School of Business and Economics, Loughborough University, Loughborough, Leicestershire, LE11 3TU, UK

HIGHLIGHTS

- A new decomposition of total factor productivity (TFP) growth into own and spillover components is proposed.
- Our decomposition is applied using a spatial autoregressive frontier model.
- Production frontier analysis of 40 European countries for the period 1995–2008.
- The 2004 EU accession countries reduce the average efficiency of EU countries.
- Returns to scale spillovers make a positive contribution to a country's TFP growth.

ARTICLE INFO

Article history:
Received 16 October 2012
Received in revised form
27 February 2013
Accepted 2 March 2013
Available online 13 March 2013

JEL classification:
C23
C51
D24

Keywords:
Spatial autoregression
Frontier modeling
Panel data
Total factor productivity (TFP) growth

ABSTRACT

A new spatial decomposition of total factor productivity growth into direct (own) and indirect (spillover) components is set out. We then apply the decomposition in the context of a spatial autoregressive production frontier analysis of 40 European countries over the period 1995–2008.

1. Introduction

Frontier modeling is a useful empirical framework with which to analyze total factor productivity (TFP) growth. Typically, the three components of TFP growth (own efficiency change, own technological change, and own scale change) are computed from a fitted frontier model. This approach is commonly used to analyze TFP growth of countries (e.g. Kumar and Russell, 2002; Kumbhakar and Wang, 2005). However, it is increasingly being recognized that economic growth and development are spatially dependent. In this note, we extend the above standard TFP growth decomposition to include direct (own) and indirect (spillover) components using a spatial autoregressive production frontier model.[1] We then apply our spatial TFP growth decomposition using data for 40 European countries over the period 1995–2008.

2. The spatial autoregressive production frontier model

We apply the method proposed by Cornwell et al. (1990), in which unit-specific effects are used to calculate time-varying efficiencies, to a model in which there is spatial autoregressive dependence. We do not discuss spatial panel data models in detail here, but for a comprehensive and up-to-date survey see Baltagi (2011). Along the above lines, (1) represents a spatial autoregressive

* Corresponding author. Tel.: +44 1509 222704; fax: +44 1509 223910.
E-mail address: A.J.Glass@lboro.ac.uk (A. Glass).

[1] A direct component in our spatial TFP growth decomposition differs from that in the standard non-spatial TFP growth decomposition. This is because our direct components contain feedback effects, i.e. effects which pass through other units via the spatial multiplier matrix and back to the unit which initiated the change. The standard non-spatial TFP growth decomposition overlooks these feedback effects. Our extension of the standard non-spatial TFP growth decomposition is based on an indirect component being interpreted as a spillover to the *i*th unit from all the other units in the sample.

0165-1765/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.econlet.2013.03.002

production frontier model:

$$y_{it} = \kappa + \alpha_i + \tau_t + TL\,(x,t)_{it} + \lambda \sum_{j=1}^{N} w_{ij} y_{jt} + z_{it}\phi + \varepsilon_{it},$$

$$i = 1, \ldots, N;\ t = 1, \ldots, T. \quad (1)$$

N is a cross-section of units; T is the fixed time dimension; y_{it} is the output of the ith unit; α_i is a unit-specific fixed effect; τ_t is a time period effect; $TL\,(x,t)_{it}$ represents the technology as the translog approximation of the log of the production function, where x is a vector of inputs; λ is the spatial autoregressive parameter; w_{ij} is an element of the spatial weights matrix, W; z_{it} is a vector of exogenous characteristics and ϕ is the associated vector of parameters; and ε_{it} is an i.i.d. disturbance for i and t with zero mean and variance σ^2.

W is a row-normalized $(N \times N)$ matrix of known positive constants which describes the spatial arrangement of the cross-sectional units and also the strength of the spatial interaction between the units. All the elements on the main diagonal of W are set to zero. λ is assumed to lie in the interval $(1/r_{min}, 1)$, where r_{min} is the most negative real characteristic root of W, and, because W is row normalized, 1 is the largest real characteristic root of W.[2] We model the effects of time in (1) by, first, including a time trend, t, and the associated quadratic and cross terms in the translog function and, second, via time period effects to account for common macroeconomic shocks.[3]

We estimate (1) using maximum likelihood, and account for the endogeneity of the spatial autoregressive variable and the fact that ε_t is not observed by including the scaled logged determinant of the Jacobian transformation of ε_t to y_t (i.e. $T \log |I_N - \lambda W|$) as a term in the log-likelihood function. Details of the estimation of (1) by demeaning in the space dimension can be found in Elhorst (2009) with the following caveat. Lee and Yu (2010) show that demeaning in the space dimension to estimate a model such as (1) results in a biased estimate of σ^2 when N is large and T is fixed, which we denote σ_B^2. Following Lee and Yu (2010), we correct for this bias by replacing σ_B^2 in the variance matrix with the bias-corrected estimate of σ^2, $\sigma_{BC}^2 = T\sigma_B^2/(T-1)$, which changes the t-values.[4]

We calculate time-varying efficiencies using the 'modifying' estimation procedure (Cornwell et al., 1990). Summarizing, using the residuals from (1), ε_{it}, we estimate $\varepsilon_{it} = \theta_i t + \rho_i t^2 + e_{it}$, where e_{it} is an i.i.d. disturbance. We then use the θ_i and ρ_i parameters together with the α_i parameters, which are retrieved using the estimate of (1), to calculate the technical efficiencies using (2). In each period, it is assumed that the most efficient unit lies on the frontier:

$$TE_{it} = \exp\left(\left(\alpha_i + \theta_i t + \rho_i t^2\right) - \max_i \left(\alpha_i + \theta_i t + \rho_i t^2\right)\right). \quad (2)$$

3. Marginal effects and spatial TFP growth decomposition

LeSage and Pace (2009) demonstrate that for a model such as (1) the coefficients on the explanatory variables cannot be interpreted as elasticities. This is because the marginal effect of an explanatory variable is a function of the spatial autoregressive variable. LeSage and Pace (2009) therefore suggest the following approach to calculate direct, indirect, and total marginal effects. Stacking successive cross-sections, we can rewrite (1) as

$$\begin{aligned} y_t = {} & (I_N - \lambda W)^{-1} \kappa \iota_N + (I_N - \lambda W)^{-1} \alpha_N \\ & + (I_N - \lambda W)^{-1} \tau_t \iota_N + (I_N - \lambda W)^{-1} \Gamma_t \beta \\ & + (I_N - \lambda W)^{-1} z_t \phi + (I_N - \lambda W)^{-1} \varepsilon_t, \end{aligned} \quad (3)$$

where ι_N is an $(N \times 1)$ vector of ones; α_N is an $(N \times 1)$ vector of fixed effects; Γ_t is an $(N \times K)$ matrix of stacked observations for $TL\,(x,t)_t$; and β is a vector of translog parameters.

Differentiating (3) with respect to the kth variable in $TL\,(x,t)_t$, $\Gamma_{k,t}$, yields the following vector of partial derivatives:

$$\left[\frac{\partial y}{\partial \Gamma_{k,1}} \cdot \frac{\partial y}{\partial \Gamma_{k,N}}\right]_t = \begin{bmatrix} \frac{\partial y_1}{\partial \Gamma_{k,1}} & \cdot & \frac{\partial y_1}{\partial \Gamma_{k,N}} \\ \cdot & \cdot & \cdot \\ \frac{\partial y_N}{\partial \Gamma_{k,1}} & \cdot & \frac{\partial y_N}{\partial \Gamma_{k,N}} \end{bmatrix}_t = (I_N - \lambda W)^{-1} \begin{bmatrix} \beta_k & 0 & \cdot & 0 \\ 0 & \beta_k & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \beta_k \end{bmatrix}, \quad (4)$$

where the product of the matrices on the far right of (4) is independent of time. The main diagonal of this product consists of direct effects, and all the non-diagonal elements of this product are indirect effects. Since (4) yields different direct and indirect effects on each unit, LeSage and Pace (2009) suggest reporting a mean direct effect (average of the diagonal elements of the product of matrices on the far right of (4)) and a mean indirect effect (in our case the average row sum of the non-diagonal elements from the same product). The direct effect which includes feedback effects is the mean effect of changing an independent variable in a cross-sectional unit on that unit's dependent variable. The indirect effect which we report is the mean change in the dependent variable for one particular unit following a change in an independent variable in all the other units. The mean total effect is the sum of the mean direct and indirect effects. We calculate the t-statistics for the mean effects directly by obtaining the standard errors using the delta method.[5]

As noted by Kumar and Russell (2002), the standard three components of productivity change ($\dot{TFP}$) are change in own technical efficiency (EC), own technical change (TC), and change in own returns to scale (SC) ($\dot{TFP} = EC + TC + SC$). We extend this standard non-spatial decomposition of $\dot{TFP}$ to include both direct and indirect TC and SC components. By making use of the quadratic identity lemma (Caves et al., 1982), the following expression for $\dot{TFP}$ with direct and indirect TC and SC components can be obtained.

$$\begin{aligned} \dot{TFP}_{it+1} = {} & \underbrace{[\ln TE_{it+1} - \ln TE_{it}]}_{EC} + \underbrace{1/2\left[\eta t_{it+1}^{Dir} + \eta t_{it}^{Dir}\right]}_{\text{Direct TC}} + \underbrace{1/2\left[\eta t_{it+1}^{Ind} + \eta t_{it}^{Ind}\right]}_{\text{Indirect TC}} \\ & + \underbrace{1/2\left[\sum_{r=1}^{R}\left(\left(\eta x_{r,it+1}^{Dir} SF_{it+1}^{Dir}\right) + \left(\eta x_{r,it}^{Dir} SF_{it}^{Dir}\right)\right) \ln\left(x_{r,it+1}^{Dir}/x_{r,it}^{Dir}\right)\right]}_{\text{Direct SC}} \\ & + \underbrace{1/2\left[\sum_{r=1}^{R}\left(\left(\eta x_{r,it+1}^{Ind} SF_{it+1}^{Ind}\right) + \left(\eta x_{r,it}^{Ind} SF_{it}^{Ind}\right)\right) \ln\left(x_{r,it+1}^{Ind}/x_{r,it}^{Ind}\right)\right]}_{\text{Indirect SC}}, \end{aligned} \quad (5)$$

2 Furthermore, $(I_N - \lambda W)$ is taken to be non-singular for all values of λ in the parameter space. It is also assumed that the row and column sums of W and $(I_N - \lambda W)$ are bounded uniformly in absolute value. This limits the spatial correlation to a manageable degree.

3 We thank an anonymous referee for proposing that we also include time period effects.

4 We do not also demean in the time dimension even though we include time period dummies. This is because it would eliminate the time trend and the associated quadratic term, which we want to retain because of their role in the TFP growth decomposition. Not demeaning in the time dimension does not create an incidental parameter problem in the application, as the sample only spans 14 years. In the application, when estimating the corresponding non-spatial model using standard software, a small number of time period dummies are dropped by the software for reasons of collinearity. We drop the same time period dummies when fitting the spatial frontier models.

5 We thank an anonymous referee for suggesting the delta method to compute the t-statistics as an alternative to Monte Carlo simulation.