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When can we do better than autarky?

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HIGHLIGHTS

- We study risk sharing between two noncommitted agents with stochastic income.
- We provide a sufficient and necessary condition for nonautarkic contracts to exist.
- Higher patience of the agents helps to satisfy the condition.
- Higher variability of the random income helps to satisfy the condition.
- Verifying the condition takes just one Gaussian elimination of a matrix.

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ABSTRACT

This paper provides a necessary and sufficient condition for the existence of nonautarkic contract in a risk sharing model with two-sided lack of commitment. Verifying the condition takes just one Gaussian elimination of a matrix.

1. Introduction

The theory of contracting with two-sided lack of commitment has been applied to study a wide range of economic issues, including international business cycles (cf. Kehoe and Perri, 2002), consumption inequality (cf. Kocherlakota, 1996 and Ligon et al., 2002), and wage contracts (cf. Thomas and Worrall, 1988). In this theory, a commonly made assumption is that some nonautarkic risk sharing arrangement is sustainable (in the sense that no one would leave the contract). To satisfy this assumption, researchers focus on sufficiently patient economic agents, in which case a Folk-theorem argument shows that nearly any allocation is sustainable. Away from this extreme, a natural question is: Under what conditions does a nonautarkic and sustainable risk sharing arrangement exist?

To answer this question, we study agents' incentives to participate in risk sharing. We linearize their utilities around autarkic endowment, which allows us to calculate in closed form the cost and the benefit of participation. Hence the condition for participation is simply that the benefit exceeds the cost. Besides answering the above question, the analysis of the linearized model provides clear economic insights on agents' incentives that are difficult to identify in the original nonlinear model.

2. Model

The model is similar to that in Ligon et al. (2002). There are two agents at time zero, with preferences

$$E\left[\sum_{t=0}^{\infty}\delta^t u(c_t^1)\right] \quad \text{and} \quad E\left[\sum_{t=0}^{\infty}\delta^t v(c_t^2)\right],$$

where c_t^i $(i = 1, 2)$ is agent i's consumption at time t, $\delta \in (0, 1)$ is their common discount factor, and E is the expectation operator. Both agents are risk averse, i.e., $u'' < 0$, $v'' < 0$. In each period t, agent i's income y_i depends on the state of the nature s, which is drawn from a finite set $\{1, 2, \ldots, S\}$ and follows a Markov chain. Let Π be the transition matrix $[\pi_{sr}]_{s,r=1}^S$, where π_{sr} is the transition

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probability from state s to state r. We assume $\pi_{sr} > 0$ for all s and r to simplify the analysis.

A risk sharing contract specifies for each t and each history $h_t \equiv (s_0, s_1, \ldots, s_t)$ a transfer $\tau(h_t)$ to be made from agent 1 to agent 2. Transfers can be negative. Neither agent can commit; if one defaults, then both of them go to autarky (i.e., transfers are zero thereafter). Conditional on h_t, the expected surplus of agent 1 over autarky is

$$U(h_t) \equiv u(y_1(s_t) - \tau(h_t)) - u(y_1(s_t)) + E\sum_{j=t+1}^{\infty} \delta^{j-t}\left(u(y_1(s_j) - \tau(h_j)) - u(y_1(s_j))\right),$$

and the surplus of agent 2, $V(h_t)$, is defined similarly. A contract is *sustainable* if $U(h_t) \geq 0$ and $V(h_t) \geq 0$, for all h_t. All contracts to be discussed in this paper are sustainable.

A sustainable contract is *(constrained) efficient* if for any given level of agent 1's surplus it provides more surplus to agent 2 than other sustainable contracts. Ligon et al. (2002) show that, if nonautarkic contracts exist, then an efficient contract is characterized as follows. There exist $\{\bar{U}_s > 0\}_{s=1}^S$ and agent 2's surplus functions $\{V_s(\cdot) : [0, \bar{U}_s] \to \mathbb{R}\}_{s=1}^S$ such that

$$V_s(U_s) = \max_{\tau_s, \{U_r\}_{r=1}^S} v(y_2(s) + \tau_s) - v(y_2(s)) + \delta \sum_{r=1}^S \pi_{sr} V_r(U_r) \quad (1)$$

$$\text{subject to} \quad u(y_1(s) - \tau_s) - u(y_1(s)) + \delta \sum_{r=1}^S \pi_{sr} U_r = U_s, \quad (2)$$

$$U_r \in [0, \bar{U}_r].$$

The surplus function $V_s(U_s)$ decreases in U_s and reaches zero at $U_s = \bar{U}_s$.

2.1. A linearized problem

Following Thomas and Worrall (1988) and Ligon et al. (2002), this subsection considers a model with utilities linearized around autarkic endowment. We show below that the linearized model not only is analytically more tractable, it also offers clear intuition about the cost and the benefit of participating in this long-term contract. Fix $\{\bar{U}_s > 0\}_{s=1}^S$ in problem (1). Suppose agents' utilities in state s are $u(y_1(s)) + u'(y_1(s))(c^1 - y_1(s))$ and $v(y_2(s)) + v'(y_2(s))(c^2 - y_2(s))$. Agent 2's problem is

$$\mathbb{L}_s(U_s) = \max_{\tau_s, \{U_r\}_{r=1}^S} v'(y_2(s))\tau_s + \delta \sum_{r=1}^S \pi_{sr} \mathbb{L}_r(U_r)$$

$$\text{subject to } -u'(y_1(s))\tau_s + \delta \sum_{r=1}^S \pi_{sr} U_r = U_s, \quad U_r \in [0, \bar{U}_r].$$

Introducing $c_s \equiv u'(y_1(s))\tau_s$, $\xi_s \equiv \frac{v'(y_2(s))}{u'(y_1(s))}$, $A_r \equiv -U_r$, and $L_r(A_r) \equiv \mathbb{L}_r(U_r)$, we rewrite the above as

$$L_s(A_s) = \max_{c_s, \{A_r\}_{r=1}^S} \xi_s c_s + \delta \sum_{r=1}^S \pi_{sr} L_r(A_r) \quad (3)$$

$$\text{subject to } c_s + \delta \sum_{r=1}^S \pi_{sr} A_r = A_s, \quad A_r \in [-\bar{U}_r, 0]. \quad (4)$$

Without loss of generality, we assume that the ratio of marginal utilities ξ_s weakly increases in s.

Problem (3) has the following interpretation. Both agent 1 and 2 have linear utilities and their consumptions are $-c_s$ and c_s, respectively. Agent 2 is subject to taste shocks $\{\xi_s\}_{s=1}^S$ while agent 1 is not. Because of the taste shocks, agent 2 prefers consumption in states with high ξ_s while agent 1 is indifferent. To facilitate trade, agent 2 opens a "bank account" with agent 1, in which agent 2's asset holding A_s represents how much agent 1 owes agent 2. Noncommitment of agent 1 requires $A_s \leq 0$ (i.e., agent 2 is in debt) at all times: positive A_s would obligate agent 1 to repay and trigger his default. On the other hand, although agent 2 is in debt, he would not default as long as he can still benefit from trading with agent 1. To see the benefit, interpret (4) as agent 2's budget constraint. There are two channels through which agent 2 can move consumptions from low-taste-shock states to high-taste-shock states: (1) he can reallocate assets among future states, holding more assets in high-shock states; and (2) when the current taste shock is high, agent 2 can increase his consumption through borrowing (i.e., holding less assets in the future). Calculating these benefits is the key to understanding agent 2's default decision; the following lemma does this in closed form.

Lemma 1. *$L_s(A_s) = L_s(0) + \xi_s A_s$, where*

$$L(0) = \delta(I - \delta\Pi)^{-1} B\hat{U}, \quad (5)$$

$$L(0) \equiv \begin{pmatrix} L_1(0) \\ L_2(0) \\ \vdots \\ L_S(0) \end{pmatrix},$$

$$B \equiv \begin{pmatrix} 0 & 0 & \cdots & 0 \\ \pi_{21}(\xi_2/\xi_1 - 1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{S1}(\xi_S/\xi_1 - 1) & \pi_{S2}(\xi_S/\xi_2 - 1) & \cdots & 0 \end{pmatrix},$$

$$\hat{U} \equiv \begin{pmatrix} \xi_1 \bar{U}_1 \\ \xi_2 \bar{U}_2 \\ \vdots \\ \xi_S \bar{U}_S \end{pmatrix}.$$

Proof. That $L_s(A_s)$ is linear in A_s is because agent 2's utility function is linear. To find out $L_s(0)$, note that the optimal portfolio choice in problem (3) is

$$A_r = \begin{cases} -\bar{U}_r, & \text{if } r < s; \\ 0, & \text{if } r \geq s. \end{cases}$$

Therefore, $c_s = A_s + \delta \sum_{r=1}^{s-1} \pi_{sr} \bar{U}_r$, and the Bellman equation is

$$L_s(0) = \xi_s\left(\delta \sum_{r=1}^{s-1} \pi_{sr}\bar{U}_r\right) + \delta \sum_{r=1}^{s-1} \pi_{sr}\left(L_r(0) - \xi_r \bar{U}_r\right) + \delta \sum_{r=s}^{S} \pi_{sr} L_r(0)$$
$$= \delta \sum_{r=1}^{s-1} (\xi_s - \xi_r)\pi_{sr}\bar{U}_r + \delta \sum_{r=1}^{S} \pi_{sr} L_r(0).$$

Solving the above linear system of equations yields (5). □

All elements in the matrix $(I - \delta\Pi)^{-1}$ are positive because $(I - \delta\Pi)^{-1} = \sum_{t=0}^{\infty} \delta^t \Pi^t$. This and (5) imply that $L_s(0) \geq 0$ for all s. If all ratios of marginal utilities are identical ($\xi_1 = \xi_s$ for all s), then autarky is the first best outcome. In this case, $B = 0$ and $L(0) = 0$. If there are two states with different ratios of marginal utilities, then at least one element in B is positive. Then $L_s(0) > 0$ for all s because all elements in $(I - \delta\Pi)^{-1}$ are positive.

Remark 1. $L_s(0)$ measures agent 2's benefit from trading with agent 1. Because agent 2's initial asset holding is zero, his average consumption is zero too. Hence, the benefit is purely from shifting consumptions from low to high taste-shock states.