Economics Letters 119 (2013) 354–357

Contents lists available at SciVerse ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Vacancies in supply chain networks☆

John William Hatfield [a], Scott Duke Kominers [b,*]

[a] Graduate School of Business, Stanford University, United States

[b] Becker Friedman Institute for Research in Economics, University of Chicago, United States

ARTICLE INFO

Article history:
Received 23 June 2011
Received in revised form
28 January 2013
Accepted 8 February 2013
Available online 18 February 2013

JEL classification:
C78
L14

Keywords:
Matching
Networks
Stability
Vacancy chains

ABSTRACT

We use the supply chain matching framework to study the effects of firm exit. We show that the exit of an initial supplier or end consumer has monotonic effects on the welfare of initial suppliers and end consumers but may simultaneously have positive and negative effects on intermediaries. Furthermore, we demonstrate that there are no clear comparative statics for the effects of intermediary exit on the welfare of other firms; most surprisingly, intermediary exit may diminish the welfare of other firms at the same level of the supply chain.

1. Introduction

In 2008, Ford Motor Company President and CEO Alan R. Mulally (Mulally, 2008) testified before Congress, advocating for a bailout of Ford's direct competitors General Motors and Chrysler. This behavior at first seems difficult to reconcile with economic theory—why should Ford plead for the survival of its direct competitors?[1,2] However, as we show in this note, this behavior can arise naturally when intermediate producers in supply chain networks have preferences over suppliers.[3]

We model the effect of exit from supply chain networks using the supply chain matching model of Ostrovsky (2008). We demonstrate two contrasting results: The exit of an end consumer benefits other end consumers while harming the initial suppliers at the head of the supply chain.[4] Meanwhile, there are no clear comparative statics for the welfare effects of removing an intermediate producer on initial suppliers, end consumers, and other intermediaries.[5] In particular, contrary to standard intuition regarding the loss of competitors, removing an intermediary may diminish the welfare of other firms at the same level of the supply chain.

Our results sharpen Theorem 3 of Ostrovsky (2008), which shows that when an initial supplier is removed from the market, the best and worst stable outcomes for other initial suppliers improve, while those for end consumers worsen. The Ostrovsky (2008) result only compares the extremal stable outcomes in a market with and without a given supplier. By contrast, we study

☆ We are grateful to Daron Acemoglu, Drew Fudenberg, Sonia Jaffe, Fuhito Kojima, Michael Ostrovsky, Assaf Romm, Alvin E. Roth, and workshop participants at Harvard for helpful comments. Hatfield appreciates the hospitality of Harvard Business School, which hosted him during parts of this research. Kominers gratefully acknowledges the support of an NSF Graduate Research Fellowship, NSF Grant CCF-1216095, a Yahoo! Key Scientific Challenges Program Fellowship, and a Terence M. Considine Fellowship in Law and Economics funded by the John M. Olin Center at Harvard Law School.

* Correspondence to: Department of Economics, University of Chicago, 1126 East 59th Street, Chicago, IL 60637, United States. Tel.: +1 773 702 0249; fax: +1 773 795 6891.

E-mail addresses: hatfield@stanford.edu, john.hatfield@gmail.com (J.W. Hatfield), skominers@uchicago.edu, skominers@gmail.com (S.D. Kominers).

[1] At the time there was significant concern that, without government action, General Motors and Chrysler could be forced to liquidate (Isidore, 2008). Thus, it seems likely that without government action General Motors and Chrysler would (at least) have become weaker competitors for Ford.

[2] We are indebted to Daron Acemoglu for this example. Acemoglu et al. (2012) give an alternative explanation of Ford's behavior, focusing on issues of aggregate volatility in supply chain networks.

[3] Such preferences arise whenever firm interactions involve relationship-specific capital (Williamson, 1983). Relationship-specific capital has been identified, e.g., in manufacturing (Parsons, 1972) and coal markets (Joskow, 1987).

[4] By symmetry, an analogous result holds for the effects of an initial supplier's exit.

[5] Similar analysis shows that there are no clear comparative statics for the effects of initial supplier (or end consumer) exit on intermediary welfare.

0165-1765/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
doi:10.1016/j.econlet.2013.02.004

the process of market reequilibration following firm exit; this allows us to characterize the effect of initial supplier exit on any given stable outcome.

Our work follows in the tradition of "vacancy chain" results for matching markets. We show that the vacancy chain results of Gale and Sotomayor (1985), Blum et al. (1997), and Hatfield and Milgrom (2005) generalize to supply chain networks, but only in a very specific sense—they apply only to firms at the ends of the supply chain, and not to intermediaries.[6] These observations underscore the importance of relation-specific contracting in supply chain dynamics.

2. Model

We begin by introducing the standard supply chain matching framework of Ostrovsky (2008), using the notation of Hatfield and Kominers (2012); readers familiar with matching theory may wish to skip to Section 3.

There is finite set F of firms, and a finite set X of contracts. Each contract $x \in X$ is associated with a buyer x_B and a seller x_S; there may be several contracts between the same buyer and seller. For notational convenience, we let $Y|_f \equiv \{y \in Y : f \in \{y_B, y_S\}\}$ denote the set of contracts in Y associated with firm f; we extend this notation by writing $Y|_G = \cup_{g\in G}(Y|_g)$ for $G \subseteq F$.

We assume that the contract set X is *acyclic*, i.e. that there does not exist a set of contracts

$$\{x^1, \ldots, x^N\} \subseteq X$$

such that $x_B^1 = x_S^2, x_B^2 = x_S^3, \ldots, x_B^{N-1} = x_S^N, x_B^N = x_S^1$. This assumption is equivalent to the assumption of *supply chain structure*, i.e. the existence of an ordering $\lhd$ on F such that for all $x \in X, x_S \lhd x_B$.

Preferences

Each $f \in F$ has a strict preference relation P^f over sets of contracts involving f. For any $Y \subseteq X$, the *choice set* of f is the set of contracts f chooses from Y,

$$C^f(Y) \equiv \max_{P^f}\{Z \subseteq Y : x \in Z \Rightarrow f \in \{x_B, x_S\}\}.^7$$

The purchase contracts chosen by f from $Y \subseteq X$, given access to sale contracts in $Z \subseteq X$, are recorded by

$$C_B^f(Y|Z) \equiv \{x \in C^f(\{y \in Y : y_B = f\} \cup \{z \in Z : z_S = f\}) : x_B = f\}.$$

Analogously, we define

$$C_S^f(Z|Y) \equiv \{x \in C^f(\{y \in Y : y_B = f\} \cup \{z \in Z : z_S = f\}) : x_S = f\}.$$

We also define the *rejected set* of contracts when acting as a buyer or as a seller as

$$R_B^f(Y|Z) \equiv Y - C_B^f(Y|Z),$$
$$R_S^f(Z|Y) \equiv Z - C_S^f(Z|Y).$$

Let $C_B(Y|Z) \equiv \cup_{f\in F} C_B^f(Y|Z)$ be the set of contracts chosen from Y by some firm as a buyer, and $C_S(Z|Y) \equiv \cup_{f\in F} C_S^f(Z|Y)$ be the set of contracts chosen from Z by some firm as a seller. Let $R_B(Y|Z) \equiv Y - C_B(Z|Y)$ and $R_S(Z|Y) \equiv Z - C_S(Z|Y)$.

The preferences of $f \in F$ are *same-side substitutable* if for all $Y' \subseteq Y \subseteq X$ and $Z' \subseteq Z \subseteq X$,

1. $R_B^f(Y'|Z) \subseteq R_B^f(Y|Z)$ and
2. $R_S^f(Z'|Y) \subseteq R_S^f(Z|Y)$.

Similarly, the preferences of $f \in F$ are *cross-side complementary* if for all $Y' \subseteq Y \subseteq X$ and $Z' \subseteq Z \subseteq X$,

1. $R_B^f(Y|Z) \subseteq R_B^f(Y|Z')$ and
2. $R_S^f(Z|Y) \subseteq R_S^f(Z|Y')$.

If a firm's preferences are both same-side substitutable and cross-side complementary, then the firm has *fully substitutable* preferences: the firm is more willing to enter into a contract as a buyer if either there are fewer purchase opportunities available (same-side substitutability), or there are more sale opportunities available (cross-side complementarity). Similarly, the firm is more willing to enter into a contract as a seller if either there are fewer other sale opportunities available (same-side substitutability), or there are more purchase opportunities available (cross-side complementarity).

Stability

An *outcome* is a set of contracts $A \subseteq X$. An outcome is *stable* if it is

1. *Individually rational*: for all $f \in F, C^f(A) = A|_f$;
2. *Unblocked*: there does not exist a nonempty *blocking set* $Z \subseteq X$ such that $Z \not\subseteq A$ and $Z|_f \subseteq C^f(A \cup Z)$ for all $f \in Z_F$.

Stability is the standard solution concept of matching theory (Roth and Sotomayor, 1990; Hatfield and Milgrom, 2005). In the presence of fully substitutable preferences, it is equivalent to the *chain stability* solution concept studied by Ostrovsky (2008); moreover, it is known in the presence of fully substitutable preferences that stable outcomes always exist (Ostrovsky, 2008; Hatfield and Kominers, 2012).

3. Vacancy dynamics

To formally study the effects of market exit in the supply chain matching model established above, we first introduce the following generalized deferred acceptance operator Φ^G, which tracks contract offers made after the firms in $G \subseteq F$ leave the market:

$$\Phi_B^G(X^B, X^S) \equiv X - (R_S(X^S|X^B) \cup (X|_G))$$

$$\Phi_S^G(X^B, X^S) \equiv X - (R_B(X^B|X^S) \cup (X|_G))$$

$$\Phi^G(X^B, X^S) \equiv (\Phi_B^G(X^B, X^S), \Phi_S^G(X^B, X^S)).$$

For any input (X^B, X^S) to the operator Φ^G, we say that X^B and X^S are *buyer* and *seller offer sets* associated with the outcome $X^B \cap X^S$. Note that at each iteration of Φ^G all offers made to firms in G (i.e. contracts in $(X^B \cup X^S) \cap (X|_G)$) are removed.

When firms' preferences are fully substitutable, iteration of the operator Φ^G on input (X^B, X^S) leads to a fixed point $\tilde{\Phi}^G(X^B, X^S)$. Moreover, for any fixed point (X^B, X^S) of Φ^G, the outcome $X^B \cap X^S$ associated with (X^B, X^S) is a stable outcome of the economy with firms $F - G$ and contract set $X|_{F-G}$ (Hatfield and Kominers, 2012).

We model the *exit* of firms $G \subseteq F$ from the economy as a transition from the economy with firm set F and contract set X to the economy with firm set $F - G$ and contract set $X|_{F-G}$. Following the exit of $G \subseteq F$, the dynamics of the market readjustment from a stable outcome A associated with offer sets X^B and X^S follow the running of the generalized deferred acceptance operator Φ^G starting with input $(X^B|_{F-G}, X^S|_{F-G})$; that is, following the exit of G from the economy stabilized at $A = X^B \cap X^S$, the market restabilizes at the stable outcome associated with $\tilde{\Phi}^G(X^B, X^S)$.

Under these vacancy dynamics, the impact of a firm's exit depends on that firm's position in the supply chain. To see this, we separately consider firms which are

[6] Our positive result also applies in more restricted settings for which vacancy chain results have not previously been proven, such as the settings of many-to-many matching (Echenique and Oviedo, 2006) and many-to-many matching with contracts (Klaus and Walzl, 2009; Hatfield and Kominers, 2011).

[7] Here, we use the notation $\max_{P^f}$ to indicate that the maximization is taken with respect to the preferences of firm f.